Set Theory

→ हमारे Mathematics की भाषा मे Universe मे सब कुछ या तो living या non-living एक object है।

→ दिया गया collection of object "well-defined" कहलाएगा। अगर, हम यह पक्के तौर पर यह कह सके की दिया हुआ object एक collection को belong करता है या नहीं।

→ SET: एक well defined collection of objects को कहते हैं।

→ Set मे जो objects होते है उन्हें members or elements or points कहते हैं।

→ हम sets को capital letters से denote करते है i.e. A, B, C ... etc.

→ अगर मान लो a is an element of a set A, we write $a \in A$; जिसका मतलब है a belongs to A or that a is an element of A.

→ अगर a element set A मे नही belong करता है तो उसे हम ऐसे लिखेंगे, $a \notin A$.

examples :-

(i) english alphabet के सभी vowels का collection, इस set में five elements होते हैं, namely a, e, i, o, u.

(ii) Odd natural numbers less than 10 का collection, इस set में होंगे 1, 3, 5, 7, 9.

(iii) prime numbers less than 20 का collection, इस set में होंगे 2, 3, 5, 7, 11, 13, 17, 19.

How to describe or specify a set

(1.) Roster Form or Tabulation Method :-

इस method में हम वह सारे Members को list कर देंगे set के within braces { } and सभी को commas से seperate कर देंगे।

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example

(i.) A = set of all factors of 24

$\therefore A = \{1, 2, 3, 4, 6, 8, 12, 24\}$

(ii.) B = set of all prime numbers between 50 and 70

$\therefore B = \{53, 59, 61, 67\}$

(iii.) C = set of all integers between $-\frac{3}{2}$ and $\frac{11}{2}$.

$\therefore C = \{-1, 0, 1, 2, 3, 4, 5\}$

Note :

N : Set of all natural numbers.
Z : Set of all integers.
Q : Set of all rational numbers
R : Set of all real numbers.

(2.) Set - Builder Form :-

इस Method में हम वह property list या properties जो set का हर element satisfy करता है उसे list करते हैं।

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हम लिखेंगे,

$\{x : x \text{ has properties } P\}$.

हम इसे पढ़ेंगे :-

'the set of all those x such that each x satisfies properties P'.

examples:

(i) $A = \{1, 2, 3, 4, 5, 6, 7\}$.

can be written as

$A = \{x : x \in N \text{ and } x < 8\}$

(ii) $B = \{1, 2, 4, 7, 14, 28\}$

can be written as

$B = \{x : x \in N \text{ and } x \text{ is a factor of } 28\}$

(iii) $C = \{2, 4, 8, 16, 32\}$.

साफ है की $C = \{2^1, 2^2, 2^3, 2^4, 2^5\}$

can be written as

$C = \{x : x = 2^n, \text{ where } n \in N \text{ and } 1 \le n \le 5\}$

(iv.) $D = \{-6, -4, -2, 0, 2, 4, 6\}$

साफ बात है कि D = set of even integers from -6 to 6.

This, can be written as

$$D = \{x : x = 2m, \text{ where } m \in Z \text{ and } -3 \le m \le 3\}$$

(v.) $E = \{3, 6, 9, 12, 15, 18\}$

साफ बात है

$$E = \{3 \times 1, 3 \times 2, 3 \times 3, 3 \times 4, 3 \times 5, 3 \times 6\}$$

इसलिए,

$$E = \{x : x = 3n, \text{ where } n \in N \text{ and } 1 \le m \le 6\}$$

(vi.) $F = \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \frac{8}{9}\right\}$

साफ बात है

$$F = \left\{x : x = \frac{m}{(m+1)}, \text{ where } m \in N \text{ and } 1 \le m \le 8\right\}$$

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(vii) $G = \{1, 3, 5, 7, 9, 11, \dots\}$

$G = \{x : x \in N \text{ and } x \text{ is odd}\}$

(viii) $H = \{1, 4, 9, 16, 25, 36, \dots\}$

$H = \{x : x = n^2, \text{ where } n \in N\}$

Q. Write the following sets in roster form :—

(i) $A = \{x : x \text{ is a natural no.}, 30 \leq x < 36\}$

Ans: $A = \{30, 31, 32, 33, 34, 35\}$

(ii) $B = \{x : x \text{ is an integer and } -4 < x < 6\}$

Ans: $B = \{-3, -2, -1, 0, 1, 2, 3, 4, 5\}$

(iii) $C = \{x : x$ is a two-digit number such that the sum of its digits is $9\}$

Ans: $C = \{18, 81, 27, 72, 36, 63, 45, 54, 90\}$

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(iv) $D = \{x : x$ is an integer, $x^2 \le 9\}$

Ans: $D = \{-3, -2, -1, 0, 1, 2, 3\}$

(v) $E = \{x : x$ is a prime no; which is a divisor of 42.$\}$

Ans: $E = \{2, 3, 7\}$

(vi) $F = \{x : x$ is a letter in the word 'MATHEMATICS'$\}$

Ans: $F = \{M, A, T, H, E, M, A, T, I, C, S\}$

(vii) $G = \{x : x$ is a prime number and $80 < x < 100\}$

Ans: $G = \{83, 89, 97\}$

(viii) $H = \{x : x$ is a perfect square and $x < 50\}$

Ans: $H = \{1, 4, 9, 16, 25, 36, 49\}$

(ix) $J = \{x : x \in R$ and $x^2 + x - 12 = 0\}$

Ans: $J = \{-4, 3\}$

(x.) $K = \{x : x \in N, x \text{ is a multiple of } 5 \text{ and } x^2 < 400\}$

Ans: $K = \{5, 10, 15\}$

Q. List all the elements of each of the sets given below :-

(i) $A = \{x : x = 2n, n \in N \text{ and } n \leq 5\}$

Ans: $A = \{2, 4, 6, 8, 10\}$

(ii) $B = \{x : x = 2n+1, n \in W \text{ and } n < 5\}$

Ans: $B = \{1, 3, 5, 7, 9\}$

(iii) $C = \{x : x = \frac{1}{n}, n \in N \text{ and } n < 6\}$

Ans: $C = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\}$

(iv.) $D = \{x : x = n^2, n \in N \text{ and } 2 \leq n \leq 5\}$

Ans: $D = \{4, 9, 16, 25\}$

(V.) $E = \{x : x \in Z \text{ and } x^2 = x\}$

Ans: $E = \{0, 1\}$.

Q. write each of the following in set-builder form :-

(i.) $A = \left\{1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \frac{1}{36}, \frac{1}{49}\right\}$

Ans: $A = \left\{x : x = \frac{1}{n^2}, n \in N \text{ and } 1 \leq n \leq 7\right\}$

(ii.) $B = \left\{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26}, \frac{6}{37}, \frac{7}{50}\right\}$

Ans: $B = \left\{x : x = \frac{n}{(n^2+1)}, n \in N \text{ and } 1 \leq n \leq 7\right\}$

(iii.) $C = \{53, 59, 61, 67, 71, 73, 79\}$

Ans: $C = \{x : x \text{ is prime}, 50 < x < 80\}$

(iv.) $D = \{-1, 1\}$

Ans: $D = \{x : x \in Z, x^2 = 1\}$

(v) $E = \{x : x = 7n, n \in N, 2 \leq n \leq 14\}$

Ans: $E = \{14, 21, 28, 35, 42, \ldots, 98\}$

Some Terms Related to Sets

* **Empty Set :-**
एक ऐसा set जिसमे कोई भी element नहीं हो।

⇒ इसके Null set or void set भी कहते हैं।

example :

(i) $\{x : x \in N \text{ and } 2 < x < 3\} = \phi$, since there is no natural number lying between 2 and 3.

(ii) $\{x : x \text{ is a number}, x \neq x\} = \phi$, since there is no number which is not equal to itself.

(iii) $\{x : x \in N, x < 5 \text{ and } x > 7\} = \phi$, since there is no natural no. which is less than 5 and greater than 7.

★ **Singleton Set :-** यह एक ऐसा set होता है जिसमें सिर्फ एक element होगा।

Example :

(i) $\{0\}$ is a singleton set whose only element is 0.

(ii) $\{15\}$ is a singleton set whose only element is 15.

(iii) $\{-8\}$ is a singleton set whose only element is -8.

★ **Finite Sets & Infinite Sets**

एक empty या non-empty set जिसमें no. of elements का counting एक end पर आ जाए उसे finite set.

एक ऐसा set जो finite set नहीं है उसे infinite set कहते हैं।

⇒ The no. of distinct elements contained in a finite set A is denoted by $n(A)$.

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examples of finite set :-

(i.) $A = \{2, 4, 6, 8, 10, 12\}$

साफ है की A एक finite set है
क्योंकी $n(A) = 6$

(ii.) B = set of all letters in the English Alphabet

साफ है की B एक finite set है
क्योंकी $n(B) = 26$

(iii.) $C = \{x : x \in Z, \text{ and } x^2 - 36 = 0\}$

साफ है की $C = \{-6, 6\}$, जो की एक finite set है क्योंकी $n(C) = 2$.

examples of infinite set :

(i.) The set of all points on the arc of a circle is an infinite set.

(ii.) The set of all points on a line segment is an infinite set

(iii.) The set of all straight lines parallel to a given line, say the x-axis is an infinite set

⇒ Each of the sets N, Z, Q and R is an infinite set.

NOTE :- All infinite sets cannot be described in roster form.

☆ Equal Set :-

दो non-empty sets A एवं B equal कहा जाएगा, अगर उनके पास exactly same elements हों and we write, $A = B$.

नहीं तो sets unequal हैं और हम लिखेंगे $A \neq B$.

Remarks :-

(i) The elements of a set may be listed in any order

जैसे की, $\{1, 2, 3\} = \{2, 1, 3\} = \{3, 2, 1\}$

(ii) The repetition of elements in a set has no meaning.

जैसे की, $\{1, 1, 2, 2, 3\} = \{1, 2, 3\}$

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Examples:—

(i) A = set of letters in the word 'follow'.

B = set of letters in the word 'wolf'.

Ans: साफ है कि, हमारे पास

$A = \{f, o, l, w\}$ and

$B = \{w, o, l, f\}$

$\therefore A = B$

(ii) $A = \{p, q, r, s\}$

$B = \{q, r, p, s\}$

$A \neq B$.

(iii) $A = \{x : x \in N, x^2 - 9 = 0\}$ and
$B = \{x : x \in Z, x^2 - 9 = 0\}$

Ans: $x^2 - 9 = 0 \Rightarrow (x+3)(x-3) = 0$

$\Rightarrow x = -3$ or $x = 3$

$\therefore A = \{x : x \in N, x^2 - 9 = 0\} = \{3\}$

& $B = \{x : x \in Z, x^2 - 9 = 0\} = \{-3, 3\}$

$\therefore A \neq B$

☆ Equivalent Set :-

दो finite sets A and B equivalent कहलाएँगे, अगर
$n(A) = n(B)$

⇒ equal sets are always equivalent But, equivalent sets need not be equal.

Ex1: Let $A = \{1, 3, 5\}$ and $B = \{2, 4, 6\}$

अतः $n(A) = n(B)$

इसलिए A और B equivalent हैं

साथ है कि $A \neq B$.

इसलिए A और B equivalent sets हैं equal नहीं।

Ex2: Show that $\{0\}$ and ϕ are not equivalent sets.

मान लिया कि $A = \{0\}$ &
$B = \phi$

अतः साफ है कि $n(A) = 1$ and $n(B) = 0$.

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$\therefore n(A) \neq n(B)$

और इसलिए A और B equivalent sets नहीं है।

Q. Which of the following are examples of the null set?

(i) Set of odd natural numbers divisible by 2

$\Rightarrow$ इस दुनिया में अभी तक कोई ऐसा odd natural no. नहीं है जो 2 से divisible हो।

$\rightarrow$ ~~Not~~ a null set

(ii) Set of even prime numbers.

$\Rightarrow \{2\} \rightarrow$ Not a null set

(iii) $A = \{x : x \in N, 1 < x \leq 2\}$

$\Rightarrow \{2\} \rightarrow$ Not a null set

(iv) $B = \{x : x \in N, 2x+3 = 4\}$

$\Rightarrow$ There is no natural no. which satisfies the given equation $2x+3 = 4$.

$\hookrightarrow$ null set

(v.) $C = \{x : x \text{ is prime}, 90 < x < 96\}$

$\Rightarrow$ दुनिया मे ऐसा कोई prime no. नही है जो (90, 96) के ~~a~~ interval मे lie करता हो।

$\hookrightarrow$ ~~no~~ a null set.

(vi.) $D = \{x : x \in N, x^2 + 1 = 0\}$

$\Rightarrow$ दुनिया मे ऐसा कोई natural No. नही जो इस equation को satisfy कर सके।

$\hookrightarrow$ a null set

Q. Which of the following are examples of the singleton set?

(i) $\{x : x \in Z, x^2 = 4\}$

Ans: $\{-2, 2\}$ $\rightarrow$ this is not a singleton set

(ii) $\{x : x \in Z, x + 5 = 0\}$

Ans: $\{-5\}$ $\rightarrow$ so it is a singleton set

(iii) $\{x : x \in Z, |x| = 1\}$

Ans: $\{-1, 1\}$ → this is not a singleton set.

(iv) $\{x : x \in N, x^2 = 16\}$

Ans: $\{4\}$ → this is a singleton set

(v) $\{x : x$ is an even prime no.$\}$

Ans: $\{2\}$ → this is a singleton set

Q. Which of the following are pairs of equal sets?

(i) A = set of letters in the word 'ALLOY'
B = set of letters in the word 'LOYAL'.

Ans:

$A = \{A, L, O, Y\}$

$B = \{L, O, Y, A\}$

therefore, $n(A) = n(B)$
also, the four elements of A is same as of B.
↳ A & B are equal sets

(ii) C = set of letters in the word, 'CATARACT'
D = set of letters in the word, 'TRACT'

Ans:

$C = \{C, A, T, R\}$

$D = \{T, A, R, C\}$

therefore, $n(C) = n(D)$
also,
the four elements of C is same as of D.
↳ C & D are equal sets

(iii) $E = \{x : x \in Z, x^2 \leq 4\}$

$F = \{x : x \in Z, x^2 = 4\}$

Ans: Clearly,

$E = \{-2, -1, 0, 1, 2\}$

$F = \{-2, 2\}$

$\because n(E) \neq n(F)$.

↳ E & F are not equal sets.

(iv). $G = \{-1, 1\}$

$H = \{x : x \in Z, x^2 - 1 = 0\}$

Ans:

Clearly, $G = \{-1, 1\}$

$H = \{-1, 1\}$

therefore, $n(G) = n(H) = 2$

also, elements of A is exactly same as that of B.

इसलिए, G & H are equal set.

(V.) $J = \{2, 3\}$

$K = \{x : x \in Z, (x^2 + 5x + 6) = 0\}$

Ans: Clearly,

$J = \{2, 3\}$

for K $x^2 + 5x + 6 = 0$

$\Rightarrow x^2 + 3x + 2x + 6 = 0$

$\Rightarrow x(x + 3) + 2(x + 3) = 0$

$\Rightarrow (x+3)(x+2) = 0$

$K = \{-3, -2\}$.

इसलिए, J & K are not equal set.

Q. Which of the following are pairs of equivalent sets?

(i) $A = \{-2, -1, 0\}$ and $B = \{1, 2, 3\}$

Ans: $n(A) = n(B) = 3$

A & B are equivalent set.

(ii) $C = \{x : x \in N, x < 3\}$ and

$D = \{x : x \in W, x < 3\}$.

Ans: Clearly,

$C = \{1, 2, 3\}$

$D = \{0, 1, 2, 3\}$

$n(C) = 3$ $n(D) = 4$

$\therefore n(C) \neq n(D)$

C & D are not equivalent set.

(iii) $E = \{a, e, i, o, u\}$ and

$F = \{p, q, r, s, t\}$

Ans: $n(E) = n(F) = 5.$

E & F are equivalent set.

Q. State whether the given set is finite or infinite :

(i) A = set of all triangles in a plane.

Ans: इस एक plane पर कितने भी numbers of triangle हो सकते हैं इसलिए

A is infinite set

(ii) B = set of all points on the circumference of a circle.

Ans: Circle के circumference पर कितने भी numbers of points हो सकते हैं

इसलिए

B is infinite set

(P.T.O.)

(iii) C = set of all lines parallel to the y-axis.

Ans: y-axis के parallel infinite lines हो सकती है
इसलिए
∴ C is infinite set

(iv) D = set of all leaves on a tree.

Ans: tree पर कितने पेड़ हैं आज तक किसी मानव गिन नहीं सकते
इसलिए
D is infinite set

(v) E = set of all positive integers greater than 500.

Ans: 500 से अधिक सभी positive integers अनंत (infinite) है।

Sub Sets

☆ **Subset:** एक set A set B का subset कहलाएगा अगर set A का हर element set B का भी हो या उसमे भी हो।
इसको लिखेंगे $A \subseteq B$.

☆ **Superset:** अगर $A \subseteq B$ और
Proper Subset: $A \neq B$ तब A को proper subset of B कहेंगे और हम लिखेंगे,
$A \subset B$.

☆ **Superset:** अगर $A \subseteq B$ तब B को कहेंगे superset of A और लिखेंगे

$B \supseteq A$

Remark: अगर एक single element भी A मे है जो B मे नही है तो A जो है B का subset नही होगा
और इसे लिखेंगे

$A \not\subset B$.

examples of subsets

① $A = \{2, 3, 5\}$ &
$B = \{2, 3, 5, 7, 9\}$.

$\therefore A \subseteq B$ but $A \neq B$.
इसलिए A is a proper subset of set B, i.e. $A \subset B$.

② $A = \{1, 2\}$ &
$B = \{2, 3, 5\}$

अब $1 \in A$ लेकिन $1 \notin B$

$\therefore A \not\subseteq B$.
फिर $3 \in B$ लेकिन $3 \notin A$.

$\therefore B \not\subseteq A$.

Thus $A \not\subseteq B$ and $B \not\subseteq A$.

③ $N \subset W \subset Z \subset Q \subset R$

$N = \{1, 2, 3, 4 \ldots\}$
$W = \{0, 1, 2, \ldots\}$
$Z = \{\ldots -2, -1, 0, 1, 2, 3, 4 \ldots\}$
$Q = \{x : x$ is a no. of form $\frac{p}{q}$, $p, q \in R\}$
$\mathbb{R} = (-\infty, \infty)$
clearly $N \subset W \subset Z \subset Q \subset R$.

Q. Let $A = \{1, \{2,3\}, 4\}$.
Then, which of the following statements is true?

(i) $\{2,3\} \in A$ (b) $\{2,3\} \subset A$

Rectify the wrong statement.

Ans: साफ है कि A एक set है जिसमें तीन elements हैं, namely $1, \{2,3\}$ & 4

(i) $\{2,3\} \in A$ is a true statement.

(b) $\{2,3\} \subset A$ is wrong.

इसको अगर हमको सही कर के लिखना है तो हम कुछ ऐसे लिखेंगे

$\{\{2,3\}\} \subset A$ as true statement.

Some results on subsets :—

(1) Every set is a subset of itself.
i.e. $A \subseteq A$.

कहने का मतलब यह है कि हर set खुद का subset होता है।

(2) The empty set is a subset of every set. i.e. $\phi \subset A$.

कहने का मतलब यह है कि ϕ यानी ऐसा set जिसमें कोई element नहीं होता वह सभी set का subset होता है।

(3) The total number of subsets of a set containing n elements is 2^n.

Proof के साथ समझो :—

मान लिया की A एक finite set है जिसमें n elements है ; तब,

no. of subsets of A each containing no element
$$= 1 = {}^nC_0.$$

no. of subsets of A each containing 1 element

$= {}^nC_1$

no. of subsets of A each containing 2 elements

$= {}^nC_2$

....

....

no. of subsets of A each containing n elements

$= {}^nC_n$.

$\therefore$ Total number of subsets of A.

$= ({}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n)$

$= 2^n$ {binomial theorem के knowledge से}

(P.T.O.)

✯ Universal Set :-

अगर हम बहुत सारे sets पर काम कर रहे हैं तो अगर एक ऐसा set हो जिसमें बाकी सारे set समा जाएं या बाकी सारे set इस set के subset हो तो इस set को हम universal set कहेंगे।

⇒ हम इसको "U" से denote करेंगे।

Q. Let $A = \{1, 2, 3\}$, $B = \{2, 3, 4, 5\}$
$C = \{6, 7\}$.

Ans: अगर हम consider करें एक set $U = \{1, 2, 3, 4, 5, 6, 7\}$ तो साफ है कि U superset है हर दिए हुए sets का।

इसलिए, U is the universal set.

Subsets of the set R of all Real numbers

(i) $N = \{1, 2, 3, 4, 5, \ldots\}$
is the set of all natural numbers.

(ii) $W = \{0, 1, 2, 3, 4, 5, \ldots\}$
is the set of all whole numbers.

(iii) $Z = \{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
is the set of all integers.

$Z^+ = \{1, 2, 3, 4, \ldots\}$.
is the set of all positive integer

$Z^- = \{\ldots, -4, -3, -2, -1\}$
is the set of all negative integer

(iv) $Q = \left\{x : x = \frac{p}{q}, \text{ where, } p, q \in Z \text{ and } q \neq 0\right\}$
is the set of all rational numbers.

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(v) $T = \{x : x \in \mathbb{R} \text{ and } x \notin \mathbb{Q}\}$ is the set of all irrational numbers.

Intervals

मान लो कि $a, b \in \mathbb{R}$ and $a < b$
तब हम define किए :

(i) closed interval $[a, b]$
$= \{x \in \mathbb{R},\ a \leq x \leq b\}$

{इसके कहने का मतलब यह की a से लेकर b including a & including b.}

(ii) Open interval (a, b) or $]a, b[$
$= \{x \in \mathbb{R},\ a < x < b\}$

(iii) Right Half open Interval
$[a, b[$
$= \{x \in \mathbb{R} : a \leq x < b\}$

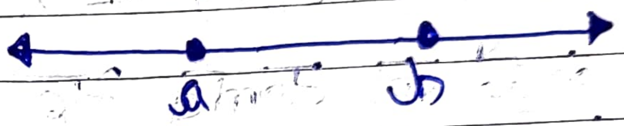
(P.T.O.)

(iv) Left Half Open Interval

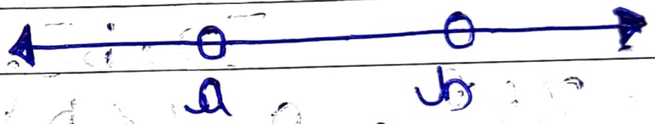
$(a, b]$ or $]a, b]$

$= \{x \in \mathbb{R} : a < x \leq b\}$

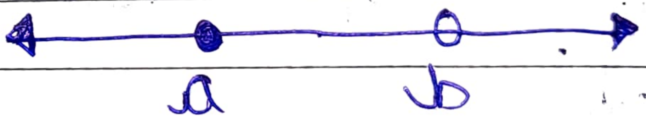
On the real line, we represent these intervals as shown below:



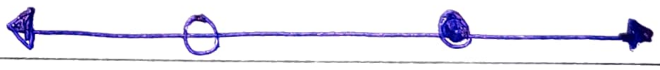
$[a, b]$



(a, b)



$[a, b)$



$(a, b]$

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* Length of an interval :

The length of each of the intervals $[a, b]$, (a, b), $[a, b)$ and $(a, b]$ is $(b-a)$.

examples :-

(i) $[-2, 3]$

$= \{x \in \mathbb{R} : -2 \leq x \leq 3\}$

(ii) $(-2, 3)$

$= \{x \in \mathbb{R} : -2 < x < 3\}$

(iii) $[-2, 3)$

$= \{x \in \mathbb{R} : -2 \leq x < 3\}$

(iv) $(-2, 3]$

$= \{x \in \mathbb{R} : -2 < x \leq 3\}$.

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✯ Power Set :-

The set of all subsets of a given set A is called the power set of A, denoted by P(A).

अगर $n(A) = m$ तब
$n[P(A)] = 2^m$.

Q. Write all possible subsets of $A = \{4\}$.

Ans: सभी possible subsets of A हैं $\phi, \{4\}$.

$\therefore P(A) = \{\phi ; \{4\}\}$

यहाँ, $n(A) = 1$

$n[P(A)] = 2 = 2^1$.

Q. Write down all possible subsets of $A = \{2, 3\}$.

Ans: सभी possible subsets of A हैं $\phi, \{2\}, \{3\}, \{2, 3\}$

$\therefore P(A) = \{\phi ; \{2\} ; \{3\} , \{2,3\}\}$

इसलिए $n(A) = 2$ &

$n\{P(A)\} = 4 = 2^2$.

Q. write down all possible subsets of $A = \{-1, 0, 1\}$.

Ans: सभी possible subsets of A.

$\phi, \{-1\}, \{0\}, \{1\}, \{-1, 0\}, \{0, 1\}, \{-1, 1\}$ and $\{-1, 0, 1\}$.

$\therefore P(A) = \{\phi, \{-1\}, \{0\}, \{1\}, \{-1, 0\}, \{0, 1\}, \{-1, 1\}, \{-1, 0, 1\}\}$

क्योंकी $n(A) = 3$

इसलिए $n[P(A)] = 8 = 2^3$.

Q. write down all possible subsets of $A = \{1, \{2,3\}\}$

Ans: यहाँ A के पास दो elements है 1 और $\{2,3\}$.

$\therefore P(A) = \{\phi, \{1\}, \{2,3\},$
$\{1, \{2,3\}\}\}$

Q. Write down all possible subsets of ϕ.

Ans: ϕ has only one subset, namely ϕ;

$$P(\phi) = \{\phi\}.$$

Q. Write each of the following intervals in the set-builder form :-

(i) $A = (2, 5)$
(ii) $B = [-4, 7]$
(iii) $C = [-8, 0)$
(iv) $D = (5, 9]$

Ans: (i) $A = (2,5) = \{x : x \in \mathbb{R}, 2 < x < 5\}$

(ii) $B = [-4, 7] = \{x : x \in \mathbb{R}, -4 \leq x \leq 7\}$

(iii.) $C = [-8, 0)$
$= \{x : x \in \mathbb{R}, -8 \leq x < 0\}$

(iv.) $D = (5, 9]$
$= \{x : x \in \mathbb{R}, 5 < x \leq 9\}$

Operations on Sets

1. Union of Sets :—

दो sets A और B का union जिसको $A \cup B$ से denote करते हैं, यह वह set है जिसमें वह सारे elements जो या तो A में हो या B में हो या A और B दोनों में हो।

ex: $A = \{3, 4, 5, 6\}$

$B = \{4, 6, 8, 10\}$

साफ है कि

$A \cup B = \{3, 4, 5, 6, 8, 10\}$

ex: $A = \{x : x$ is a prime number less than 10$\}$

and $B = \{x : x \in N, x$ is a factor of 12$\}$

साफ है कि

$A = \{2, 3, 5, 7\}$

$B = \{1, 2, 3, 4, 6, 12\}$

$\therefore A \cup B = \{1, 2, 3, 4, 5, 6, 7, 12\}$

ex: $A = \{x : x$ is a positive integer$\}$

and $B = \{x : x$ is a negative integer$\}$

साफ है कि

$A = \{1, 2, 3, ---\}$

$B = \{---- -3, -2, -1\}$

$A \cup B = \{---- -3, -2, -1, 1, 2, 3, ---\}$

Remark:

The union of n sets $A_1, A_2, A_3, \ldots,$ A_n is denoted by

$$(A_1 \cup A_2 \cup A_3 \cup \ldots\ldots \cup A_n) = \bigcup_{i=1}^{n} A_i.$$

$\boxed{2.}$ Intersection of sets:—

दो set A और B जिसको $A \cap B$ से denote करेंगे ; वह set है जिसमे वह सारा elements है जो दोनो set A & B मे common है।

इसलिए,

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

$$\therefore x \in A \cap B \Leftrightarrow x \in A \text{ and } x \in B$$

$$x \notin A \cap B \Rightarrow x \notin A \text{ or } x \notin B.$$

ex: $A = \{1, 3, 5, 7, 9\}$

$B = \{2, 3, 5, 7, 11, 13\}$

$A \cap B = \{3, 5, 7\}$

ex: $A = \{x : x \in N, x \text{ is a factor of } 12\}$

and $B = \{x : x \in N, x \text{ is a factor of } 18\}$

साध्य है कि

$A = \{1, 2, 3, 4, 6, 12\}$

$B = \{1, 2, 3, 6, 9, 18\}$

$\therefore A \cap B$

$= \{1, 2, 3, 6\}$

ex: $A = \{x : x = 3n, n \in Z\}$

$B = \{x : x = 4m, n \in Z\}$

साध्य है कि

$A = \{x : x \in Z \text{ and } x \text{ is a multiple of } 3\}$

and $B = \{x : x \in Z \text{ and } x \text{ is a multiple of } 4\}$

$\therefore A \cap B = \{x : x \in Z \text{ and } x \text{ is a multiple of both } 3 \,\&\, 4\}$

$= \{x : x \in Z \text{ and } x \text{ is a multiple of } 12\}$

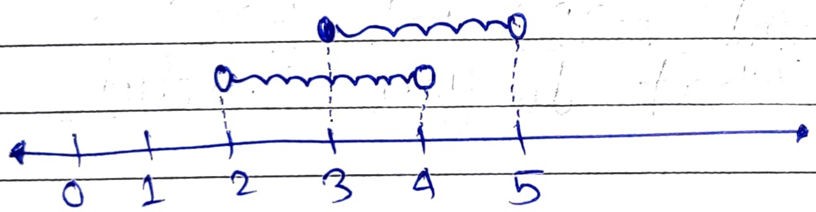
$= \{x : x = 12n, n \in Z\}$

Hence, $A \cap B = \{x : x = 12n, n \in Z\}$

Q. If $A = (2, 4)$ and $B = [3, 5)$, find $A \cap B$.

Ans: $A = (2, 4) = \{x : x \in \mathbb{R}, 2 < x < 4\}$

$B = [3, 5) = \{x : x \in \mathbb{R}, 3 \le x < 5\}$



स्पष्ट है कि

$A \cap B = \{x : x \in \mathbb{R}, 3 \le x < 4\}$

$= [3, 4)$

Remark: The intersection of n sets $A_1, A_2, A_3, \ldots\ldots, A_n$ is denoted by

$$(A_1 \cap A_2 \cap A_3 \cap \cdots \cap A_n) = \bigcap_{i=1}^{n} A_i$$

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3. **Disjoint Sets :-**
दो sets A & B को disjoint बोलेंगे अगर $A \cap B = \phi$ होगा।

4. **Intersecting Sets :-**
दो sets A & B को intersecting कहेंगे अगर $A \cap B \neq \phi$

Q. If $A = \{1, 3, 5, 7, 9\}$,
$B = \{2, 4, 6, 8\}$ and
$C = \{2, 3, 5, 7, 11\}$
find $(A \cap B)$ and $(A \cap C)$.
What do you conclude.

Ans:
$A \cap B =$

$A = \{1, 3, 5, 7, 9\}$
$B = \{2, 4, 6, 8\}$

nothing common here

$A \cap B = \phi$

अब

$A = \{1, 3, 5, 7, 9\}$
$C = \{2, 3, 5, 7, 11\}$

$A \cap C = \{3, 5, 7\}$

5. Difference of Sets :-

किसी भी Sets A और B, जिसका difference, (A−B) है उसको define करेंगे ऐसे :-

$$(A-B) = \{x : x \in A \text{ and } x \notin B\}$$

इसलिए $x \in (A-B) \Rightarrow x \in A$ and $x \notin B$

Q. If $A = \{x : x \in N, x \text{ is a factor of } 6\}$
and $B = \{x \in N : x \text{ is a factor of } 8\}$
then find :-

(i.) $A \cup B$ (ii.) $A \cap B$ (iii) $A - B$ (iv.) $B - A$.

Ans: ज्ञात है कि

$A = \{1, 2, 3, 6\}$

$B = \{1, 2, 4, 8\}$

(i.) $A \cup B = \{1, 2, 3, 4, 6, 8\}$

(ii.) $A \cap B = \{1, 2\}$

(iii.) $A - B = \{3, 6\}$

(iv.) $B - A = \{4, 8\}$

6 Symmetric Difference of Tw... Sets

दो sets A एवं B, जिसको ...
$A \Delta B$ से denote करते हैं...

$A \Delta B$ को हम define करेंगे ऐ...

$$A \Delta B = (A-B) \cup (B-A)$$

Q. Let $A = \{a, b, c, d\}$ and
$B = \{b, d, f, g\}$.

Find $A \Delta B$.

Ans: मेरे पास है

$$(A-B) = \{a, b, c, d\} - \{b, d, f, g\}$$
$$= \{a, c\}$$

$$(B-A) = \{b, d, f, g\} - \{a, b, c, d\}$$
$$= \{f, g\}.$$

$$\therefore A \Delta B = (A-B) \cup (B-A)$$
$$= \{a, c\} \cup \{f, g\}$$
$$= \{a, c, f, g\}$$

7 Complement of a Set :-

मान लिया की U एक universal set है और ये भी माना की $A \subset U$.
तब, complement of A, denoted by A' or $(U-A)$ is defined as

$$A' = \{x \in U : x \notin A\}$$

clearly, $x \in A' \Leftrightarrow x \notin A$.

Q. If $U = \{1,2,3,4,5,6,7,8\}$ and $A = \{2,4,6,8\}$,
find
(i) A' (ii) $(A')'$ ~~(iii)~~

Ans: (i) A'

$$A' = U - A$$
$$= \{1,\cancel{2},3,\cancel{4},5,\cancel{6},7,\cancel{8}\} - \{\cancel{2},\cancel{4},\cancel{6},\cancel{8}\}$$
$$= \{1,3,5,7\}.$$

(ii) $(A')'$

$$(A')' = U - A'$$
$$= \{\cancel{1},2,\cancel{3},4,\cancel{5},6,\cancel{7},8\} - \{\cancel{1},\cancel{3},\cancel{5},\cancel{7}\}$$

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$= \{2, 4, 6, 8\}$

Q let N be the universal set.

(i) If $A = \{x : x \in N$ and x is odd$\}$, find A'.

Ans: $A' = \{x : x \in N$ and x is not odd$\}$

$= \{x : x \in N$ and x is even$\}$

(ii) If $B = \{x : x \in N$, x is divisible by 3 and 5$\}$

Find B'.

Ans: $B' = \{x : x \in N$ and x is not divisible by 3 or x is not divisible by 5.$\}$

Some results on Complementation :-

If $A \subset U$

(i) $U' = \phi$

(ii) $\phi' = U$

(iii) $(A')' = A$

(iv) $A \cup A' = U$

(v) $A \cap A' = \phi$

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Laws of operation on sets :-

(i.) $A \cup A = A$ and $A \cap A = A$ [Idempotent laws]

(ii.) $A \cup \phi = A$ and $A \cap U = A$ [Identity Law]

(iii.) $A \cup B = B \cup A$ and $A \cap B = B \cap A$
[Communicative Law]

(iv.) $(A \cup B) \cup C = A \cup (B \cup C)$ and
$(A \cap B) \cap C = A \cap (B \cap C)$
[Associative Law]

(v.) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
[Distributive Law]

(vi.) $(A \cup B)' = (A' \cap B')$ [De - Morgan's Laws]

$(A \cap B)' = (A' \cup B')$

Theorem 1: (Idempotent laws)

For any set A, prove that:

(i.) $A \cup A = A$

we have

$A \cup A = \{x : x \in A \text{ or } x \in A\}$

$= \{x : x \in A\}$

$= A$

(ii.) $A \cap A = A$

we have

$A \cap A = \{x : x \in A \text{ and } x \in A\}$

$= \{x : x \in A\}$

$= A.$

Theorem 2: (Identity laws)

(i.) $A \cup \phi = A$

we have,

$$A \cup \phi = \{x : x \in A \text{ or } x \in \phi\}$$
$$= \{x : x \in A\} = A$$
$$[\because x \notin \phi]$$

(ii.) $A \cap U = A$, where U is the universal set.

we have,

$$A \cap U = \{x : x \in A \text{ and } x \in U\}$$
$$= \{x : x \in A\} = A \quad [\because x \notin \phi]$$
$$[\because A \subset U]$$

Note: ϕ and U are the identity elements for union and intersection of sets respectively.

Theorem 3: (Commutative laws)

For any two sets A and B, prove that :—

I. $A \cup B = B \cup A$

Commutative law for union of sets.

⇒ Let x be an arbitary element of $A \cup B$. Then,

$x \in A \cup B$

$\Rightarrow x \in A$ or $x \in B$
$\Rightarrow x \in B$ or $x \in A$
$\Rightarrow x \in B \cup A$

$\therefore (A \cup B) \subseteq (B \cup A)$ ------ (i)

Again, let y be an arbitrary element of $B \cup A$. Then,

$y \in B \cup A$

$\Rightarrow y \in B$ or $y \in A$
$\Rightarrow y \in A$ or $y \in B$
$\Rightarrow y \in (A \cup B)$ ------ (ii)

$\therefore (B \cup A) \subseteq (A \cup B)$ ---------- (ii)

From (i) and (ii), we get

$A \cup B = B \cup A$.

II. $A \cap B = B \cap A$

Commutative law for intersection of sets.

$\Rightarrow$ Let x be an arbitrary element of $A \cap B$. Then,

$$\begin{aligned} x \in A \cap B &\Rightarrow x \in A \text{ and } x \in B \\ &\Rightarrow x \in B \text{ and } x \in A \\ &\Rightarrow x \in B \cap A \end{aligned}$$

$\therefore (A \cap B) \subseteq (B \cap A)$ ---------- (iii)

Again, let y be an arbitrary element of $B \cap A$. Then,

$$\begin{aligned} y \in B \cap A &\Rightarrow y \in B \text{ and } y \in A \\ &\Rightarrow y \in A \text{ and } y \in B \\ &\Rightarrow y \in A \cap B \end{aligned}$$

$\therefore (B \cap A) \subseteq (A \cap B)$ ---------- (iv)

From (iii) and (iv), we get

$A \cap B = B \cap A$

THEOREM 4: (Associative law)

I. $(A \cup B) \cup C = A \cup (B \cup C)$

$\boxed{\text{Associative law for union of sets}}$

$\Rightarrow$ Let x be an arbitary element of $(A \cup B) \cup C$. Then

$x \in (A \cup B) \cup C$

$\Rightarrow x \in (A \cup B)$ or $x \in C$
$\Rightarrow (x \in A$ or $x \in B)$ or $x \in C$
$\Rightarrow x \in A$ or $(x \in B$ or $x \in C)$
$\Rightarrow x \in A$ or $x \in (B \cup C)$
$\Rightarrow x \in A \cup (B \cup C)$

$\therefore (A \cup B) \cup C \subseteq A \cup (B \cup C)$ ---- (i)

(P.T.O.)

फिर,

let y be an arbitrary element of $A \cup (B \cup C)$. Then,

$y \in A \cup (B \cup C)$

$\Rightarrow y \in A$ or $y \in (B \cup C)$
$\Rightarrow y \in A$ or ($y \in B$ or $y \in C$)
$\Rightarrow$ ($y \in A$ or $y \in B$) or $y \in C$
$\Rightarrow y \in (A \cup B)$ or $y \in C$
$\Rightarrow y \in (A \cup B) \cup C$

$\therefore A \cup (B \cup C) \subseteq (A \cup B) \cup C$ ---------- (ii)

From (i) and (ii) we get

$\therefore A \cup (B \cup C) = A \cup (B \cup C)$.

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II. $(A\cap B)\cap C = A\cap(B\cap C)$

$\boxed{\text{Associative law for intersection of sets}}$

$\Rightarrow$ Let 'x' be an arbitrary element of $(A\cap B)\cap C$. Then,

$x \in (A\cap B)\cap C$

$\Rightarrow x\in(A\cap B)$ and $x\in C$

$\Rightarrow (x\in A$ and $x\in B)$ and $x\in C$

$\Rightarrow x\in A$ and $(x\in B$ and $x\in C)$

$\Rightarrow x\in A$ and $x\in(B\cap C)$

$\Rightarrow x\in A\cap(B\cap C)$

$\therefore (A\cap B)\cap C \subseteq A\cap(B\cap C)$ — (iii)

फिर, let 'y' be an arbitrary element of $A\cap(B\cap C)$. Then,

$y\in A\cap(B\cap C)$

$\Rightarrow y\in A$ and $y\in(B\cap C)$

$\Rightarrow y\in A$ and $(y\in B$ and $y\in C)$

$\Rightarrow (y\in A$ and $y\in B)$ and $y\in C$

$\Rightarrow y\in(A\cap B)$ and $y\in C$

$\Rightarrow y\in(A\cap B)\cap C$

$\therefore A \cap (B \cap C) \subseteq (A \cap B) \cap C$ —— (iv)

from (iii) and (iv), we get

$(A \cap B) \cap C = A \cap (B \cap C)$.

Theorem 5 : (Distributive Laws)

For any three sets A, B, C prove that:

(I.) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Distributive law of union over intersection.

$\Rightarrow$ Let x be an arbitrary element of $A \cup (B \cap C)$. Then,

$x \in A \cup (B \cap C)$

$\Rightarrow x \in A$ or $x \in (B \cap C)$
$\Rightarrow x \in A$ or $(x \in B)$ and $(x \in C)$
$\Rightarrow (x \in A$ or $x \in B)$ and $(x \in A$ or $x \in C)$
[$\because$ 'or' distributes 'and']

$\Rightarrow x \in (A \cup B)$ and $x \in (A \cup C)$

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$\Rightarrow x \in (A \cup B) \cap (A \cup C)$

$\therefore A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ — (i)

Again, let y be an arbitrary element of $(A \cup B) \cap (A \cup C)$,

Then,

$y \in (A \cup B) \cap (A \cup C)$

$\Rightarrow y \in (A \cup B)$ and $y \in (A \cup C)$

$\Rightarrow (y \in A$ or $y \in B)$ and $(y \in A$ or $y \in C)$

$\Rightarrow y \in A$ or $(y \in B$ and $y \in C)$

[$\because$ 'or' distributes 'and']

$\Rightarrow y \in A$ or $y \in (B \cap C)$

$\Rightarrow y \in A \cup (B \cap C)$

$\therefore (A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$

— (ii)

(P.T.O.)

from (i) and (ii), we get :-

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

II. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Distributive law of intersection over union.

$\Rightarrow$ Let x be an arbitrary element of $A \cap (B \cup C)$. Then,

$x \in A \cap (B \cup C)$

$\Rightarrow x \in A$ and $x \in (B \cup C)$

$\Rightarrow x \in A$ and $(x \in B$ or $x \in C)$

$\Rightarrow (x \in A$ and $x \in B)$ or $(x \in A$ and $x \in C)$

[$\because$ 'and' distributes 'or']

$\Rightarrow x \in (A \cap B)$ or $x \in (A \cap C)$

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$

$\therefore A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ (iii)

(P.T.O.)

Again,

let y be an arbitrary element of $(A\cap B)\cup(A\cap C)$. Then,

$y \in (A\cap B) \cup (A\cap C)$

$\Rightarrow y \in (A\cap B)$ or $y \in (A\cap C)$

$\Rightarrow (y \in A$ and $y \in B)$ or $(y \in A$ and $y \in C)$

$\Rightarrow y \in A$ and $(y \in B$ or $y \in C)$

[$\because$ 'and' distributes 'or']

$\Rightarrow y \in A$ and $y \in (B\cup C)$

$\Rightarrow y \in A\cap(B\cup C)$.

$\therefore (A\cap B)\cup(A\cap C) \subseteq A\cap(B\cup C)$ — (iv)

from (iii) & (iv), we get

$A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$

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Theorem 6 (De-Morgan's Laws)

For any two sets A and B, prove that :-

I. $(A \cup B)' = (A' \cap B')$

Ans: Let x be an arbitrary element of $(A \cup B)'$. Then,

$x \in (A \cup B)'$

$\Rightarrow x \notin A \cup B$
$\Rightarrow x \notin A$ and $x \notin B$
$\Rightarrow x \in A'$ and $x \in B'$
$\Rightarrow x \in (A' \cap B')$

$\therefore (A \cup B)' \subseteq (A' \cap B')$ —— (i)

Again,

Let y be an arbitrary element of $(A' \cap B')$. Then,

$y \in (A' \cap B')$

$\Rightarrow y \in A'$ and $y \in B'$
$\Rightarrow y \notin A$ and $y \notin B$
$\Rightarrow y \notin (A \cup B)$

$\Rightarrow y \in (A \cup B)'$

$\therefore (A' \cap B') \subseteq (A \cup B)'$ —— (ii)

from (i) & (ii), we get

$$(A \cup B)' = (A' \cap B').$$

II. $(A \cap B)' = (A' \cup B')$.

Ans: Let x be an arbitrary element of $(A \cap B)'$. Then,

$x \in (A \cap B)'$

$\Rightarrow x \notin (A \cap B)$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \in A'$ or $x \in B'$

$\Rightarrow x \in (A' \cup B')$

$\therefore (A \cap B)' \subseteq (A' \cup B')$ —— (iii)

(P.T.O.)

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Again:

let y be an arbitrary element of $(A' \cup B')$. Then,

$y \in (A' \cup B')$

$\Rightarrow y \in A'$ or $y \in B'$

$\Rightarrow y \notin A$ or $y \notin B$

$\Rightarrow y \notin (A \cap B)$

$\Rightarrow y \in (A \cap B)'$

$\therefore (A' \cup B') \subseteq (A \cap B)'$ ——— (iv)

from (iii) & (iv), we get

$(A \cap B)' = (A' \cup B')$.

Theorem 7:
For any two sets A and B, prove that:

I. $A \subseteq B \Rightarrow B' \subseteq A'$

Ans: Let $A \subseteq B$ be given & let x be an arbitrary element of B'. Then,

$x \in B' \Rightarrow x \notin B$
$\Rightarrow x \notin A, \quad [\because A \subseteq B]$
$\Rightarrow x \in A'$

$\therefore B' \subseteq A'$

Hence, $A \subseteq B \Rightarrow B' \subseteq A'$

II. $A - B = (A \cap B')$

Ans: Let x be an arbitrary element of $(A-B)$. Then,

$x \in (A-B)$

$\Rightarrow x \in A$ and $x \notin B$
$\Rightarrow x \in A$ and $x \in B'$
$\Rightarrow x \in (A \cap B')$

$\therefore (A-B) \subseteq (A \cap B')$ —— (i)

Again,

Let y be an arbitrary element of $(A \cap B')$. Then,

(P.T.O.)

$x \in (A \cap B')$
$\Rightarrow x \in A$ and $x \in B'$
$\Rightarrow x \in A$ and $x \notin B$
$\Rightarrow x \in (A-B)$ ——— (ii)

$\therefore (A \cap B') \subseteq (A-B)$

Hence, from (i) and (ii), we get
$(A-B) = (A \cap B')$.

III. $(A-B) \cap B = \phi$

Ans: If possible,
let $(A-B) \cap B = \phi$ and
let $(A-B) \cap B$. Then,

$x \in (A-B) \cap B$
$\Rightarrow x \in (A-B)$ and $x \in B$
$\Rightarrow (x \in A$ and $x \notin B)$ and $x \in B$
$\Rightarrow x \in A$ and $(x \notin B$ and $x \in B)$
लेकिन $x \notin B$ and $x \in B$ can never hold simultaneously.

Thus, we arrive at a contradict

Since, the contradiction arrives
assuming that $(A-B) \cap B \neq \phi$
and hence

$$(A-B) \cap B = \phi.$$

IV. $(A-B) \cup (B-A) = (A \cup B) - (A \cap B)$

Ans: Let 'x' be an arbitrary element of $(A-B) \cup (B-A)$. Then,

$x \in (A-B) \cup (B-A)$

$\Rightarrow x \in (A-B)$ or $x \in (B-A)$
$\Rightarrow (x \in A$ and $x \notin B)$ or $(x \in B$ and $x \notin A)$

$\Rightarrow (x \in A$ or $x \in B)$ and $(x \notin A$ or $x \notin B)$

$\Rightarrow x \in (A \cup B)$ and $x \notin (A \cap B)$
$\Rightarrow x \in \{(A \cup B) - (A \cap B)\}$

$\therefore (A-B) \cup (B-A) \subseteq \{(A \cup B) - (A \cap B)\}$ —— (iii)

again,

let 'y' be an arbitrary element of $(A \cup B) - (A \cap B)$. Then,

$y \in (A \cup B) - (A \cap B)$.

$\Rightarrow y \in (A \cup B)$ and $y \notin (A \cap B)$

$\Rightarrow (y \in A$ or $y \in B)$ and $(y \notin A$ or $y \notin B)$

$\Rightarrow (y \in A$ and $y \notin B)$ or $(y \in B$ and $y \notin A)$

$\Rightarrow y \in (A - B)$ or $y \in (B - A)$

$\Rightarrow y \in (A-B) \cup (B-A)$. —— (iv)

from (iii) & (iv), we get

$(A-B) \cup (B-A) = (A \cup B) - (A \cap B)$.

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V. $(A-B) = A \Leftrightarrow A \cap B = \phi$

Ans: मान लो कि
$(A-B) = A$ given है
& हमें $A \cap B = \phi$ show करना है।

अगर possible हो तो :-
मान लिया कि $A \cap B \neq \phi$ &
मान लिया कि $x \in A \cap B$. Then,

$x \in A \cap B \Rightarrow x \in A$ and $x \in B$
$\Rightarrow x \in (A-B)$ and $x \in B$
$[\because A = (A-B)$ (given)$]$
$\Rightarrow (x \in A$ and $x \notin B)$ and $x \in B$
$\Rightarrow x \in A$ and $(x \notin B$ and $x \in B)$

But, $x \notin B$ and $x \in B$ both can never hold simultaneously.
Thus, we arrive at a contradiction.

Since the contradiction arises by assuming that $A \cap B \neq \phi$, so
$A \cap B = \phi$.

Thus, $(A-B) = A \Rightarrow A \cap B = \phi$

Again,
let $(A \cap B) = \phi$ and we have to show that ~~$A \cap B \neq \phi$, so~~ $(A-B) = A$

Now,
$x \in (A-B)$
$\Rightarrow x \in A$ and $x \notin B$
$\Rightarrow x \in A$ (surely)

$\therefore (A-B) \subseteq A$ ———— (v)

Again, $y \in A \Rightarrow y \notin B$ $[\because A \cap B = \phi]$

$\Rightarrow y \in A$ and $y \notin B$
$\Rightarrow y \in (A-B)$

$\therefore A \subseteq (A-B)$ ———— (vi)

Thus from (v) & (vi), we get $(A-B) = A$

$\therefore A \cap B = \phi \Rightarrow (A-B) = A$

Hence,
$(A-B) = A \Leftrightarrow (A \cap B) = \phi$.

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Q. If $(A \cup B) = (A \cap B)$ then prove that $A = B$.

Ans: Let $(A \cup B) = (A \cap B)$ be given.

Let x be an arbitrary element of
Then,

$x \in A$
$\Rightarrow x \in A \cup B$ $\quad [\because A \subseteq A \cup B]$
$\Rightarrow x \in A \cap B$ $\quad [\because A \cup B = A \cap B]$
$\Rightarrow x \in A$ and $x \in B$.

$\Rightarrow x \in B$ (surely)

$\therefore A \subseteq B$ ------------ (i)

Again, let $y \in B$. Then,

$y \in B \Rightarrow y \in A \cup B$ $\quad [\because B \subseteq A \cup B]$

$\Rightarrow y \in A \cap B$ $\quad [\because A \cup B = A \cap B]$

$\Rightarrow y \in A$ and $y \in B$.

$\Rightarrow y \in A$ (surely)

$\therefore B \subseteq A$ ------------ (ii)

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Thus, from (i) and (ii), we get

$A = B$.

Q. If $A \subseteq B$ then for any set C, prove that $(C-B) \subseteq (C-A)$.

Ans: Let $A \subseteq B$ be given.

Let $x \in (C-B)$. Then,

$\Rightarrow x \in C$ and $x \notin B$
$\Rightarrow x \in C$ and $x \notin A$ $[\because A \subseteq B]$
$\Rightarrow x \in (C-A)$.

$\therefore (C-B) \subseteq (C-A)$

Hence,

$A \subseteq B \Rightarrow (C-B) \subseteq (C-A)$.

Q. For any sets A and B, prove that:–

(i) $A \cup (A \cap B) = A$.

$\Rightarrow$ since $(A \cap B) \subseteq A$

we have, $A \cup (A \cap B) = A$

$[\because X \subseteq Y \Rightarrow X \cup Y = Y]$

(ii.) $A \cap (A \cup B) = A$

$\Rightarrow$ Since $A \subseteq (A \cup B)$,
we have

$$A \cap (A \cup B) = A$$

$[\because X \subseteq Y \Rightarrow X \cap Y = X]$

Q. For any sets A and B, prove that:

(i) $(A \cap B) \cup (A - B) = A$.

Ans: we have,

$(A \cap B) \cup (A - B)$

$= (A \cap B) \cup (A \cap B')$

$[\because A - B = (A \cap B')]$

$= A \cap (B \cup B')$

$[\because B \cup B' = U]$

$= A \cap U$

$= A$

Hence,

$(A \cap B) \cup (A - B) = A.$

(ii) $A \cup (B - A) = (A \cup B)$

→ we have

$A \cup (B - A)$

$= A \cup (B \cap A')$ $[\because B - A = (B \cap A')]$

$= (A \cup B) \cap (A \cup A')$ [by distributive law]

$= (A \cup B) \cap U$

$= (A \cup B)$

Hence,

$A \cup (B - A) = (A \cup B).$

Q. If $A \cap B' = \phi$ then prove that $A = A \cap B$ and hence show that $A \subseteq B$.

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Ans: Let $A \cap B' = \phi$ be given. Then,

$A = (A \cap U)$, where U is the universal

$= A \cap (B \cup B')$ $[\because B \cup B' = U]$

$= (A \cap B) \cup (A \cap B')$

$= (A \cap B) \cup \phi$

$= (A \cap B)$

Hence, $A = (A \cap B)$

Further,

Let, $A = A \cap B$ and let $x \in A$. Then,

$x \in A \Rightarrow x \in A \cap B$

$\Rightarrow x \in A$ and $x \in B$

$\Rightarrow x \in B$ (surely)

$\therefore A \subseteq B$.

(P.T.O.)

Q. If A, B and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$ then prove that $B = C$.

Ans: Let $A \cup B = A \cup C$ and
$A \cap B = A \cap C$

$\Rightarrow (A \cup B) \cap B = (A \cup C) \cap B$ and
$(A \cup B) \cap C = (A \cup C) \cap C$

$\Rightarrow B = (A \cap B) \cup (C \cap B)$ and
$(A \cap C) \cup (B \cap C) = C$

$\{\because B \subseteq (A \cup B)$ and $C \subseteq (A \cup C)\}$

$\Rightarrow B = (A \cap B) \cup (B \cap C)$ and
$(A \cap B) \cup (B \cap C) = C$

$\{\because A \cap C = A \cap B\}$

$\Rightarrow B = C$

Hence,

$B = C$.

Q. For any sets A, B and C, prove that :-

(i.) $A-(B\cup C) = (A-B)\cap(A-C)$

Ans: we have

$A-(B\cup C) = A\cap(B\cup C)'$

$\{\because X-Y = X\cap Y'\}$

$= A\cap(B'\cap C')$

$\{$by de-morgan's law$\}$

$= (A\cap B')\cap(A\cap C')$

$= (A-B)\cap(A-C)$

$\therefore A-(B\cup C) = (A-B)\cap(A-C)$.

(ii.) $A-(B\cap C) = \cancel{A\cap(B\cap C)'}$ $(A-B)\cup(A-C)$

Ans: we have

$A-(B\cap C) = A\cap(B\cap C)'$

$\{\because X-Y = X\cap Y\}$

$= A\cap(B'\cup C')$

$\{$by de-morgan's law$\}$

$= (A \cap B') \cup (A \cap C')$

{by distributive law}

$= (A - B) \cup (A - C)$

$\therefore A - (B \cap C) = (A - B) \cup (A - C)$

(iii) $(A \cup B) - C = \cancel{(A \cup B) \cap C'}\ (A - B) \cup (A - C)$

Ans: We have,

$(A \cup B) - C = (A \cup B) \cap C'$

$[\because X - Y = X \cap Y']$

$= (A \cap C') \cup (B \cap C')$

[by distributive law]

$= (A - C) \cup (B - C)$

$[\because X \cap Y' = X - Y]$

(iv) $(A \cap B) - C = (A - C) \cap (B - C)$

Ans: we have,

$(A \cap B) - C = (A \cap B) \cap C'$

$\{\because X - Y = X \cap Y'\}$

$= (A \cap C') \cap (B \cap C')$
$= (A - C) \cap (B - C)$

$\{\because X \cap Y' = X - Y\}$

$\therefore (A \cap B) - C = (A - C) \cap (B - C)$

Q. Let A and B be sets. If $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X, show that $A = B$.

Ans:

$\because A \cup X = B \cup X$ (given)

$\Rightarrow A \cap (A \cup X) = A \cap (B \cup X)$
$\Rightarrow (A \cap A) \cup (A \cap X) = (A \cap B) \cup (A \cap X)$
(by distributive law)

$\Rightarrow A \cup \phi = (A \cap B) \cup \phi$

$\Rightarrow A = (A \cap B)$

$\Rightarrow A \subseteq B$ ---------- (i)

$\{\because A \cap B = A \Rightarrow A \subseteq B\}$

Again, $A \cup X = B \cup X$

$\Rightarrow B \cap (A \cup X) = B \cap (B \cup X)$

$\Rightarrow (B \cap A) \cup (B \cap X) = (B \cap B) \cup (B \cap X)$

{by distributive law}

$\Rightarrow (A \cap B) \cup \phi = B \cup \phi$

$\left\{ \because B \cap X = \phi \text{ and } B \cap A = A \cap B \right\}$

$\Rightarrow A \cap B = B$

$\Rightarrow B \subseteq A$ —— (ii)

$\{\because A \cap B = B \Rightarrow B \subseteq A\}$

from (i) & (ii), we get

$A = B$.

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Q Show that the following four conditions are equivalent :-

(i.) $A \subset B$
(ii.) $A - B = \phi$
(iii.) $A \cup B = B$
(iv.) $A \cap B = A$

Ans: In order to prove the required result, we will show that;

(i) $\Rightarrow$ (ii.) $\Rightarrow$ (iii.) $\Rightarrow$ (iv) $\Rightarrow$ (i)

Now, (i.) $\Rightarrow$ (ii.)

Let $A \subset B$ be given
Then, there is no element of A which is not in B.

$\therefore A - B = \{x : x \in A \text{ and } x \notin B\} = \phi$

[$\because$ there is no element of A which is not in B.]

Hence,

$A \subset B \Rightarrow A - B = \phi$ and
therefore (i) $\Rightarrow$ (ii.)

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(ii) $\Rightarrow$ (iii):

Let $A-B=\phi$ be given. Then,

$$A-B=\phi \Rightarrow \text{every element of } A \text{ is in } B.$$
$$\Rightarrow A \subseteq B$$
$$\Rightarrow A\cup B = B.$$

Thus, $A-B=\phi \Rightarrow A\cup B=B$ and therefore.

(ii) $\Rightarrow$ (iii)

(iii) $\Rightarrow$ (iv):

Let $A\cup B=B$ be given. Then,

$$A\cup B=B \Rightarrow A\subset B \Rightarrow A\cap B=A$$

Thus,

$$A\cup B=B \Rightarrow A\cap B=A$$

and therefore,

(iii) $\Rightarrow$ (iv)

(iv) $\Rightarrow$ (i):

Let $(A\cap B)=A$ be given. Then,

$$x\in A = x\in A\cap B \quad [\because A=A\cap B]$$

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$\Rightarrow x \in A$ and $x \in B$
$\Rightarrow x \in B$ (surely).

$\therefore A \subseteq B$

Thus, $A \cap B = A \Rightarrow A \subseteq B$ and therefore; (iv.) $\Rightarrow$ (i.)

$\therefore$ (i) $\Rightarrow$ (ii.) $\Rightarrow$ (iii.) $\Rightarrow$ (iv.) $\Rightarrow$ (i.)

Hence the given four conditions are equivalent.

Q. For any sets A and B, prove that
$$P(A \cap B) = P(A) \cap P(B).$$

Ans: Let $X \in P(A \cap B)$. Then,

$X \in P(A \cap B)$
$\Rightarrow X \subseteq A \cap B$
$\Rightarrow X \subseteq A$ and $X \subseteq B$
$\Rightarrow X \in P(A)$ and $X \in P(B)$
$\Rightarrow X \in P(A) \cap P(B)$ ---------- (i)

$\therefore P(A \cap B) \subseteq P(A) \cap P(B)$

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Again, let $Y \in P(A) \cap P(B)$. Then,

$Y \in P(A) \cap P(B)$

$\Rightarrow Y \in P(A)$ and $Y \in P(B)$.
$\Rightarrow Y \subseteq A$ and $Y \subseteq B$
$\Rightarrow Y \subseteq A \cap B$
$\Rightarrow Y \in P(A \cap B)$

$\therefore P(A) \cap P(B) \cap P(A \cap B)$ —— (ii)

from (i) & (ii), we get

$P(A \cap B) = P(A) \cap P(B)$.

Q. For any two sets A and B, prove that:

$P(A) \cup P(B) \subset P(A \cup B)$

But,
$P(A \cup B)$ is not necessarily a subset of $P(A) \cup P(B)$.

Ans: let X be an arbitrary element of $P(A) \cup P(B)$. Then,

$X \in P(A) \cup P(B)$

$\Rightarrow X \in P(A)$ or $X \in P(B)$
$\Rightarrow X \subset A$ or $X \subset B$.

$\Rightarrow X \subset (A \cup B)$

$\Rightarrow X \in P(A \cup B)$

$\therefore P(A) \cup P(B) \subset P(A \cup B)$

However,

$P(A \cup B) \subset P(A) \cup P(B)$ is not always true.

For example,

let $A = \{1\}$ and $B = \{2\}$.

Then, $A \cup B = \{1, 2\}$

$\therefore P(A) = \{\phi, \{1\}\}$,

$P(B) = \{\phi, \{2\}\}$

and,

$P(A \cup B) = \{\phi, \{1\}, \{2\}, \{1,2\}\}$

Also,

$P(A) \cup P(B) = \{\phi, \{1\}, \{2\}\}$

$\therefore P(A \cup B) \not\subset P(A) \cup P(B)$.

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Hence, in general,

$$P(A \cup B) \neq P(A) \cup P(B)$$

Q. If $P(A) = P(B)$, prove that $A = B$.

Ans: let $P(A) = P(B)$. Then,

$A \subseteq A \Rightarrow A \in P(A)$

$[\because P(A) = P(B)]$

$\Rightarrow A \in P(B)$

$\Rightarrow A \subseteq B$ —— (i)

again, $B \subseteq B \Rightarrow B \in P(B)$

$\Rightarrow B \in P(A)$ $[\because P(B) = P(A)]$

$\Rightarrow B \subseteq A$ —— (ii)

from (i) & (ii), we get

$A \subseteq B$ and $B \subseteq A$

Hence

$A = B$.

Venn Diagrams

→ In order to express the relationship among sets in perspective, we represent them pictorially by means of diagrams called "Venn diagrams".

→ In these diagrams :—

* Universal set is represented by a rectangular region.

* Subsets are represented by circles inside the rectangle.

* disjoint sets are represented by disjoint circles

* intersecting sets by intersecting circles.

VENN DIAGRAMS IN DIFFERENT SITUATIONS

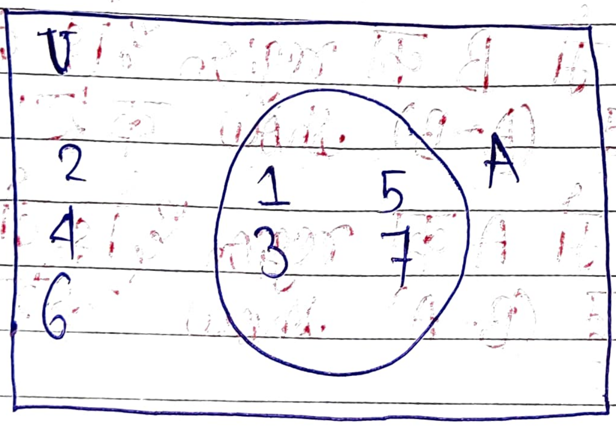
Case 1 : When the universal set and its subset are given.

मान लिया कि U universal set है &
let $A \subseteq U$.

हम एक circle draw किए एक rectangle के अंदर

जो rectangular region है वह U को represent करता है और जो circular region है वह A को represent करता है।

ex: Let $U = \{1, 2, 3, 4, 5, 6, 7\}$ &
$A = \{1, 3, 5, 7\}$



अब हम ऊपर दिए गए तरीके से venn diagram बनाएँ

और

$A' = \{2, 4, 6\}$.

Case 2 : When two intersecting subsets of U are given.

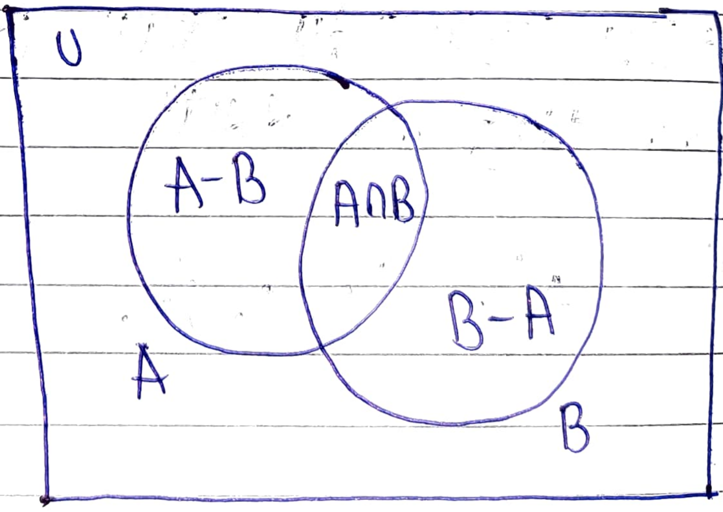
दो intersecting subsets A & B of U को represent करने के लिए हम दो intersecting circle बनाएँ rectangle के अंदर

इन circles का common region represent करता है (A∩B).

A मे B का region छोड़ दो तो वह (A−B) show करेगा।

B मे A का region छोड़ दो तो वह (B−A) show करेगा।

Ex:



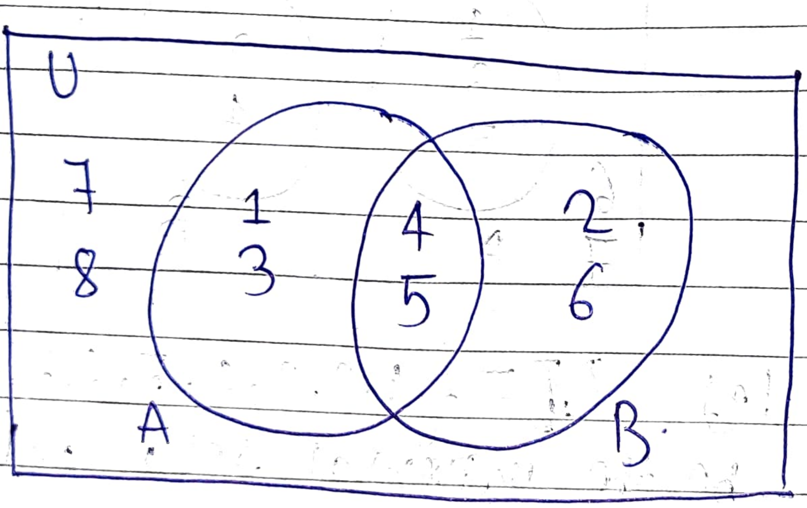
ex:

Let $U = \{1,2,3,4,5,6,7,8\}$ be the universal set, and let

$A = \{1,3,4,5\}$ & $B = \{2,4,5,6\}$ be its subsets.

तो, $A \cap B = \{4,5\}$

we draw the Venn diagram, as shown in the given figure



Clearly,

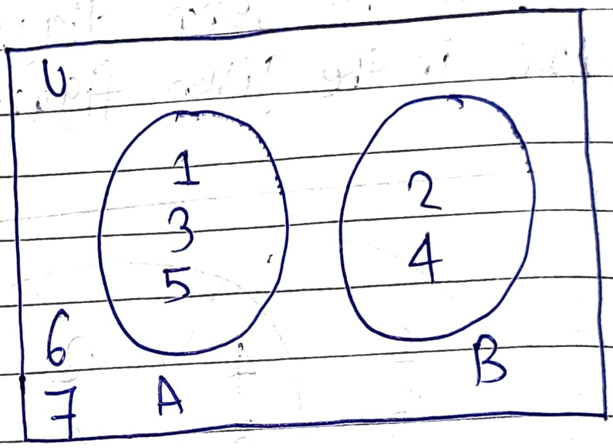
$(A-B) = \{1,3\}$ &

$(B-A) = \{2,6\}$.

Case 3: When two disjoint subsets of a set be given.

A और B दो disjoint subset एक universal set U को represent करने के लिए हम दो disjoint circles बनाएंगे एक rectangle के अंदर।

ex:



let $U = \{1, 2, 3, 4, 5, 6, 7\}$ be the universal set, and

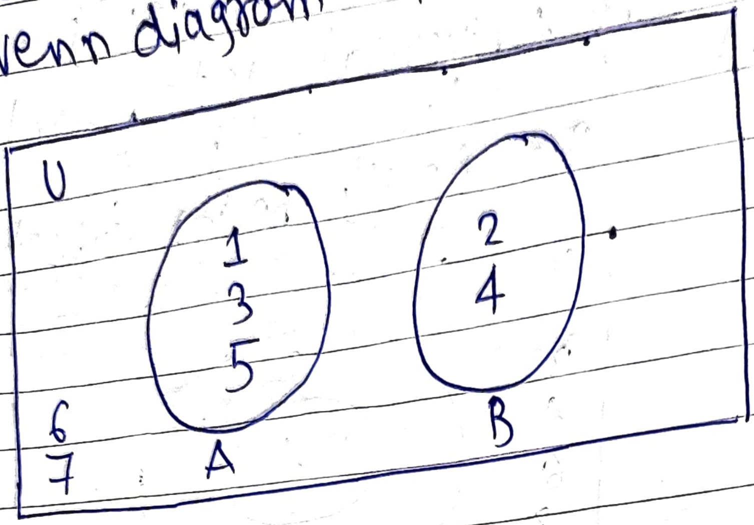
let $A = \{1, 3, 5\}$ &

$B = \{2, 4\}$

be two of its disjoint subsets

clearly, $A \cap B = \phi$

इसलिए हम नीचे दिए गए तरीके से venn diagram बनाएंगे।



Clearly,

$A \cap B = \phi$, $(A-B) = \{1,3,5\} = A$

and $(B-A) = \{2,4\} = B$

$A' = \{2,4,6,7\}$ &

$B' = \{1,3,5,6,7\}$.

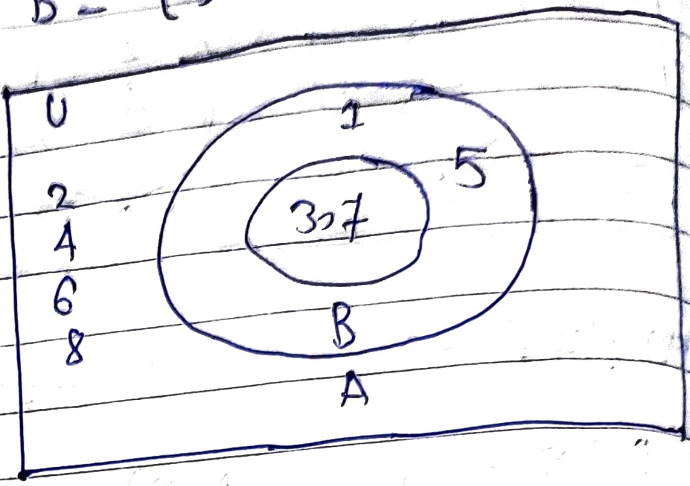
Case 4: When $B \subseteq A \subseteq U$.

In this case, we draw two concentric circles within a rectangular region.

The inner circle represents B and the outer circle represents A.

ex:

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ be the universal set, and let $A = \{1, 3, 5, 7\}$ & $B = \{3, 7\}$ be its subsets.



Then, clearly $B \subseteq A$.

अब हम venn diagram बनाएँगे

$A \cap B = B = \{3, 7\}$

$A \cup B = A = \{1, 3, 5, 7\}$.

$(A - B) = \{1, 5\}$

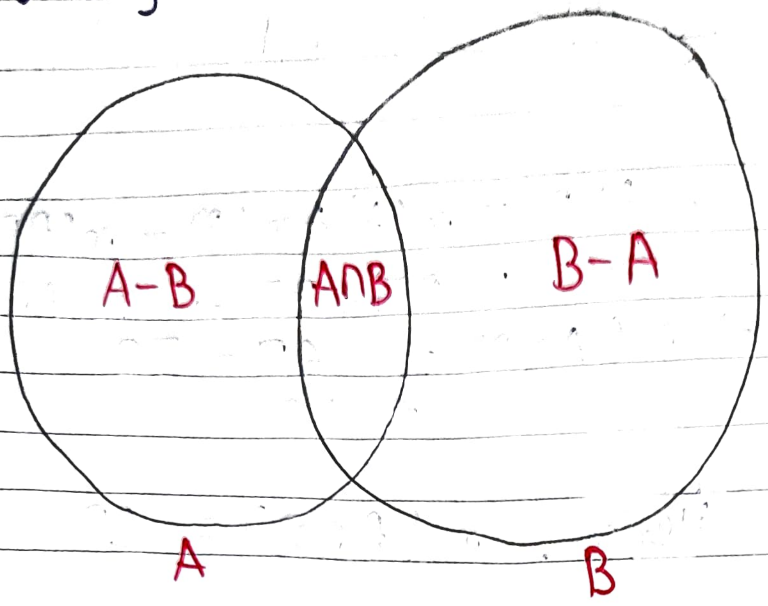
$(B - A) = \phi$.

$A' = \{2, 4, 6, 8\}$

& $B' = \{1, 5, 2, 4, 6, 8\}$

Some results derived from Venn Diagrams

For any sets A, B, C, we have :



(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) If $A \cap B = \phi$, then $\cancel{n(A}$
$n(A \cup B) = n(A) + n(B) \cancel{- n(A \cap B)}$

(iii) $n(A-B) + n(A \cap B) = n(A)$

(iv) $n(B-A) + n(A \cap B) = n(B)$

(v) $n(A \cup B \cup C) =$

$\{n(A) + n(B) + n(C) + n(A \cap B \cap C)\}$

$- \{n(A \cap B + n(B \cap C) + n(A \cap C)\}$

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Q. If A and B are two sets such that $n(A) = 27$; $n(B) = 35$ and $n(A \cup B) = 50$, find $n(A \cap B)$.

Ans:

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow 50 = 27 + 35 - n(A \cap B)$

$\Rightarrow n(A \cap B) = 62 - 50$

$= 12.$

Hence, $n(A \cap B) = 12$.

Q. If A and B are two sets containing 3 and 6 elements respectively, what can be the maximum no. of elements in $A \cup B$? Find also the minimum no. of elements in $A \cup B$?

Ans:

$\because n(A \cup B) = n(A) + n(B) - n(A \cap B)$ — (i)

Case I

from (i), it is clear that $n(A \cup B)$ will be maximum when $n(A \cap B) = 0$

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In that case,

$$n(A \cup B) = n(A) + n(B)$$
$$= 3 + 6 = 9$$

$\therefore$ maximum number of elements in $(A \cup B) = 9$.

Case II

from (i), it is clear that $n(A \cup B)$ will be minimum when $n(A \cap B)$ is maximum i.e., when $n(A \cap B) = 3$.

In this case,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$= (3 + 6 - 3) = 6$$

Hence,

Minimum no. of elements in $(A \cup B) = 6$.

Q. A survey shows that 73% of the Indians like apples, whereas 65% like oranges. What percentange of Indians like both apples and oranges?

Ans: Let,

A = set of Indians who like apples.

B = set of Indians who like oranges.

$n(A) = 73$, $n(B) = 65$

and

$n(A \cup B) = 100$

$n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 73 + 65 - 100$

$= 38$.

Hence,

38% of the Indians like both apples and oranges.

Q. In a survey of 425 students in a school, it was found that 115 drink apple juice, 160 drink orange juice and 80 drink both apple as well as orange juice. How many drink neither apple juice nor orange juice?

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Ans: Let,

U = set of all students surveyed.
A = set of all students who drink apple juice.
B = set of all students who drink orange juice.

Then,

$n(U) = 425$
$n(A) = 115$
$n(B) = 160$
$n(A \cap B) = 80$

$$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$= 115 + 160 - 80$$
$$= 275 - 80$$
$$= 195.$$

Set of students who drink neither apple juice nor orange juice

$$= (A' \cap B')$$
$$= (A \cup B)'$$

$$\Rightarrow n\{(A \cup B)'\} = n(U) - n(A \cup B)$$
$$= 425 - 195$$
$$= 230.$$

Hence,
230 students drink neither apple juice nor orange juice.

Q. In a group of 850 persons, 600 can speak Hindi and 340 can speak Tamil. Find :-

(i) how many can speak both Hindi and Tamil.
(ii) how many can speak Hindi only.
(iii) how many can speak Tamil only.

Ans: Let,

A = set of persons who can speak Hindi.

B = set of persons who can speak Tamil.

So,

$n(A) = 600$, $n(A \cup B) = 850$

$n(B) = 340$

(i) Set of persons who can speak both Hindi and Tamil

$= (A \cap B)$

Now,

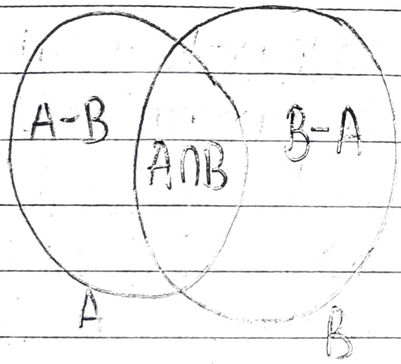
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= (600 + 340 - 850)$

$= 90.$

Thus, 90 persons can speak both Hindi and Tamil.

(ii) Set of persons who speak Hindi only. $= (A - B)$.



Now,

$n(A-B) + n(A \cap B) = n(A)$

$\Rightarrow n(A-B) = n(A) - n(A \cap B)$

$= (600 - 90)$

$= \underline{510}.$

Thus, 510 persons can speak Hindi only.

(iii) Set of persons who can speak Tamil only.

$= (B - A)$

Now,

$n(B-A) + (A \cap B) = n(B)$

$\Rightarrow n(B-A) = n(B) - n(A \cap B)$

$= 340 - 90$

$= 250$

Hence, 250 persons can speak Tamil only.

Q. A market research group conducted a survey of 1000 consumers and reported that 745 consumers like product A and 430 consumers like product B. What is the least number that must have liked both products?

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Ans: Let,
'P' be the set of consumers who like product A.
'Q' be the set of consumers who like product B.

Then, $m(P) = 745$ and $m(Q) = 430$

Now,
$m(P \cup Q) = m(P) + m(Q) - m(P \cap Q)$
$\Rightarrow m(P \cup Q) = 745 + 430 - m(P \cap Q)$
$\Rightarrow m(P \cup Q) = 1175 - m(P \cap Q)$
साफ है कि $m(P \cap Q)$ least तब होगा जब $m(P \cup Q)$ maximum होगा।
इसलिए, $m(P \cup Q) = 1000$

So,
$\Rightarrow 1000 = 1175 - m(P \cap Q)$
$\Rightarrow m(P \cap Q) = 1175 - 1000$
$= 175.$

$\therefore m(P \cap Q) = 175.$

Hence, the least no. of consumers liking both the products is 175.

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Q. Out of 600 car owners investigated, 500 owned car A; 200 owned car B and 50 owned both A and B cars. Verify whether the given data is correct or not.

Ans: Let,

P be the set who own car A

Q be the set who own car B.

Then,

$$m(P) = 500$$

$$m(Q) = \cancel{50}\ 200$$

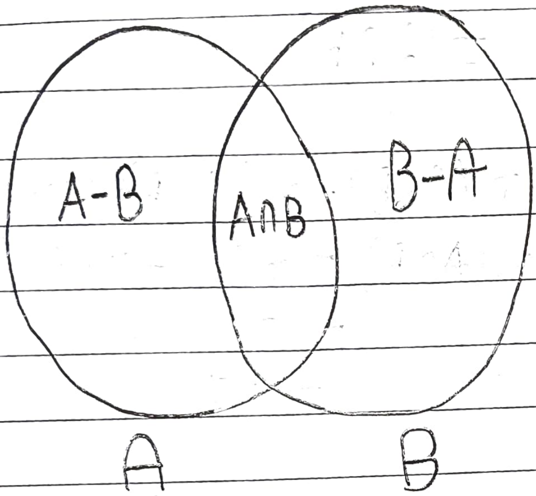
$$m(P \cap Q) = 50.$$

Now,

$$m(P \cup Q) = m(P) + m(Q) - m(P \cap Q)$$
$$= 500 + 200 - 50$$
$$= 700 - 50$$
$$= 650.$$

कुल 600 थे पर यहाँ 650 आया मतलब incorrect data.

Q. In a group of 52 persons, 16 drink tea but not coffee and 33 drink tea.
Find :-
(i) how many drink tea and coffee both.
(ii) how many drink coffe but not tea.

Ans:


माना लिया कि
A = set of persons who drink tea.
B = set of persons who drink coffee.

Then,
(A-B) = set of persons who drink tea but not coffee.

and,
(B-A) = set of persons who drink coffee but not tea.

दिया हुआ है कि :-

$n(A \cup B) = 52$
$n(A-B) = 16$
$n(A) = 33$.

(i) set of persons who drink tea and coffee both.

$= (A \cap B)$

Now,
$n(A-B) + n(A \cap B) = n(A)$
$\Rightarrow n(A \cap B) = n(A) - n(A-B)$
$= 33 - 16$
$= 17$.

(ii) Persons who drink coffee but not tea.

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow n(B) = n(A \cup B) + n(A \cap B) - n(A)$
$= 52 + 17 - 33$
$= 36$

Now,
$n(B-A) + n(A \cap B) = n(B)$
$\Rightarrow n(B-A) = n(B) - n(A \cap B)$
$= 36 - 17$
$= 19$.

Thus, number of persons who drink coffee but not tea $= 19$.

Q. A school awarded 58 medals in three sports, namely 38 in football ; 15 in basketball and 20 in cricket. If 3 students got medals in all the three sports, how many received medals in exactly two sports?

Ans: Let A → set of students who won medals in football.
B → set of students who won medals in basketball.
C → set of students who won medals in cricket.

Then,

$n(A) = 38$, $n(B) = 15$

$n(C) = 20$, $n(A \cap B \cap C) = 3$

& $n(A \cup B \cup C) = 58$.

हमें पता है कि :—

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$\Rightarrow n(A \cap B) + n(B \cap C) + n(A \cap C)$
$= n(A) + n(B) + n(C) + n(A \cap B \cap C) - n(A \cup B \cup C)$

$= \{(38 + 15 + 20 + 3) - 58\}$

$= 76 - 58$
$= 18.$

Q In a committee, 50 people speak Hindi, 20 speak English and 10 speak both Hindi and english. How many speak at least one of these two languages?

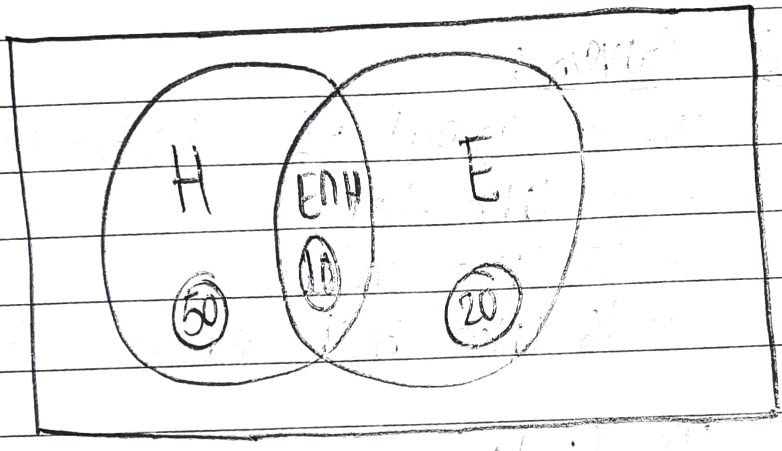
Ans: दिया हुआ :

People who speak Hindi $= 50$
People who speak English $= 20$
People who speak both English and Hindi $= 10$

To find: people who speak at least one of these two languages.

<u>let us consider,</u>

People who speak Hindi $= n(H) = 50$
People who speak English $= n(E) = 20$
People who speak both English and Hindi
$= n(H \cap E) = 10$.
People who speak atleast one of the two languages
$= n(H \cup E)$



Now, we know that,

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
therefore

$n(H \cup E) = n(H) + n(E) - n(H \cap E)$

$= 50 + 20 - 10$
$= 60$.

Thus, People who speak at least one of the two languages are 60.

Q. In a group of 50 persons, 30 like tea, 25 like coffee and 16 like both. How many like

(i.) either tea or coffee?
(ii.) neither tea nor coffee?

Ans: Given:
In a group of 50 persons,
- 30 like tea
- 25 like coffee
- 16 like both tea & coffee

To find:

(i.) People who buy either tea or coffee.

Let us consider,

Total no. of people $= n(X) = 50$
People who likes tea $= n(T) = 30$
People who likes coffee $= n(C) = 25$
People who like both tea and coffee $= n(T \cap C) = 16$

(i.) People who like either tea or coffee

$= n(T \cup C)$

$\therefore$ $n(T \cup C) = n(T) + n(C) - n(T \cap C)$

$= 30 + 25 - 16$

$= 55 - 16$

$= 39$.

(ii.) <u>People who like neither tea nor coffee.</u>

People who like neither tea nor coffee

$= n(X) - n(T \cup C)$

$= 50 - 39$

$= 11$

Therefore, People who like neither tea nor coffee $= 11$.

Q. There are 200 individuals with a skin disorder, 120 had been exposed to the chemical C_1, 50 to chemical C_2, and 30 to both the chemicals C_1 and C_2. Find the no. of individual exposed to :-

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(i.) Chemical C_1 but not chemical C_2.

(ii.) Chemical C_2 but not chemical C_1.

(iii.) Chemical C_1 or chemical C_2.

Ans: Given:

Total no. of individuals with skin disorder $= 200$

Individuals exposed to chemical C_1: $= 120$.

Individuals exposed to chemical C_2: $= 50$

Individuals exposed to chemicals C_1 and C_2 both. $= 30$.

(i.) Individuals exposed to chemical C_1 but not C_2.:

let us consider,

Total no. of individuals with skin disorder

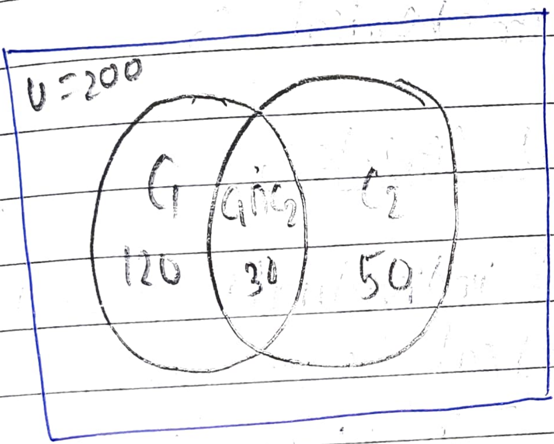
$= n(C)$

$= 200$.

Individuals exposed to chemical C_1
$= n(C_1) = 120$.

Individuals exposed to chemical C_2
$= n(C_2) = 50$

Individuals exposed to chemical C_1 and C_2 both
$= n(C_1 \cap C_2) = 30$



Individuals exposed to chemical C_1 but not C_2
$= n(C_1 - C_2)$

Now,

$n(C_1 - C_2) = n(C_1) - n(C_1 \cap C_2)$
$= 120 - 30$
$= 90$

Therefore, no. of individuals exposed to chemical C_1 but not C_2
$= 90$.

(ii) Individuals exposed to chemical C_1 or chemical C_2:

Let us consider,

No. of individuals exposed to chemical C_2 but not C_1.

$= 20$

(iii) Individuals exposed to chemical C_1 or chemical C_2.

Let us consider,

No. of individuals exposed to chemical C_1 or chemical C_2

$= n(C_1 \cup C_2)$.

Now,

$n(C_1 \cup C_2) = n(C_1) + n(C_2) - n(C_1 \cap C_2)$

$= 120 + 50 - 30$

$= 140$.

Therefore,

Number of individuals exposed to chemical C_1 or C_2

$= 140$.

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Q. In a class of a certain school, 50 students, offered mathematics, 42 offered biology and 24 offered both the subjects. Find the no. of students offering :-

(i.) mathematics only
(ii.) biology only
(iii.) any of the two subjects.

Ans: Given:

Number of students offered Maths. = 50
Number of students offered Biology = 42
No. of students offered bot Mathematics and Biology = 24.

(i) No. of students offered Maths only.

Let us consider,
No. of students offered Maths = $n(M) = 50$
No. of students offered Biology = $n(B) = 42$
No. of students offered Maths & Biology both = $n(M \cap B) = 24$.

No. of students offered Mathematics only
$= n(M - B)$

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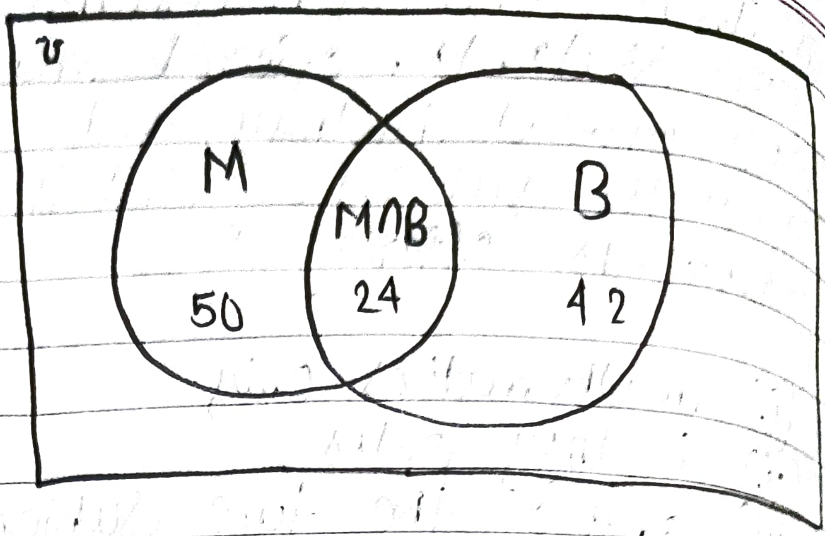

Now,

$m(M-B) = m(M) - m(M \cap B)$
$= 50 - 24$
$= 26$

(ii.) No. of students offered Biology only.

No. of students offered Biology only $= m(B-M)$

Now,

$m(B-M) = m(B) - m(M \cap B)$
$= 42 - 24$
$= 18$

Therefore, No. of students offered Biology only $= 18$.

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(iii) <u>No. of students whom offered any of the two subjects.</u>

No. of students offered any of two subjects $= n(M \cup B)$

<u>Now,</u>

$n(M \cup B) = n(M) + n(B) - n(M \cap B)$
$= 50 + 42 - 24$
$= 140$

Therefore, Number of students offered any of two subjects = 68.

Q. In an examination, 56% of the candidates failed in English and 48% failed in science. If 18% failed in both English and science, find the percentage of those who passed in both the subjects.

<u>Ans:</u> <u>Given:</u>

In an examination
- 56% of candidates failed in English.
- 48% of candidates failed in science.
- 18% of candidates failed in both English and science.

To find :-

Percentage of students who passed in both subjects.

Let us consider,

Percentage of candidates who failed in English
$= n(E)$
$= 56$

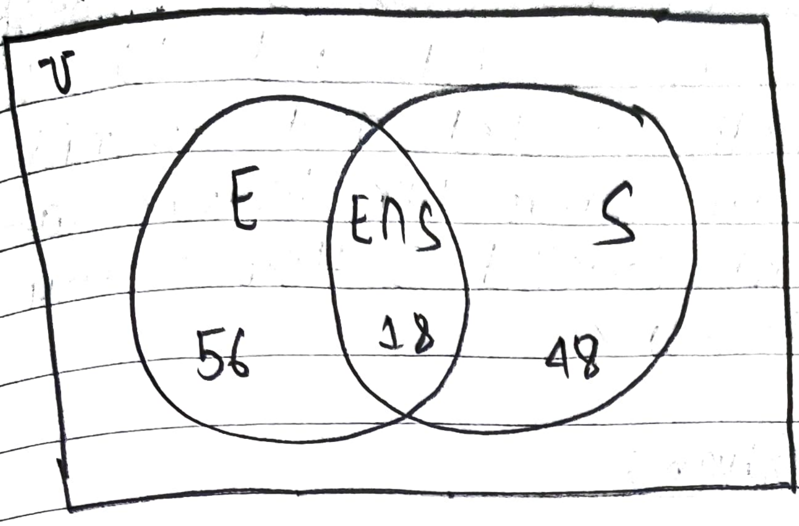
Percentage of candidates who failed in Science
$= n(S)$
$= 48$

Percentage of candidates who failed in English and science both

$= n(E \cap S)$
$= 18$

Percentage of candidates who failed in English only
$= n(E - S)$

Percentage of candidates who failed in Science only
$= n(S - E)$.



Now,

$m(E-S) = m(E) - m(E \cap S)$
$= 56 - 18$
$= 38$

$m(S-E) = m(S) - m(S \cap E)$
$= 48 - 18$
$= 30$

Therefore,

Percentage of total candidates who failed

$= m(E-S) + m(S-E) + m(E \cap S)$
$= 38 + 30 + 18$
$= 86\%$

Hence,
The percentage of candidates who passed in both English & Science $= 14\%$.

Q. In a group of 65 people, 40 like cricket and 10 like both cricket and tennis. How many like tennis and not cricket? How many like tennis?

Ans: Given:

In a group of 65 people
- 40 people like tennis only.
- 10 people like both cricket and tennis.

To find:
- Number of people like tennis only.
- Number of people like tennis.

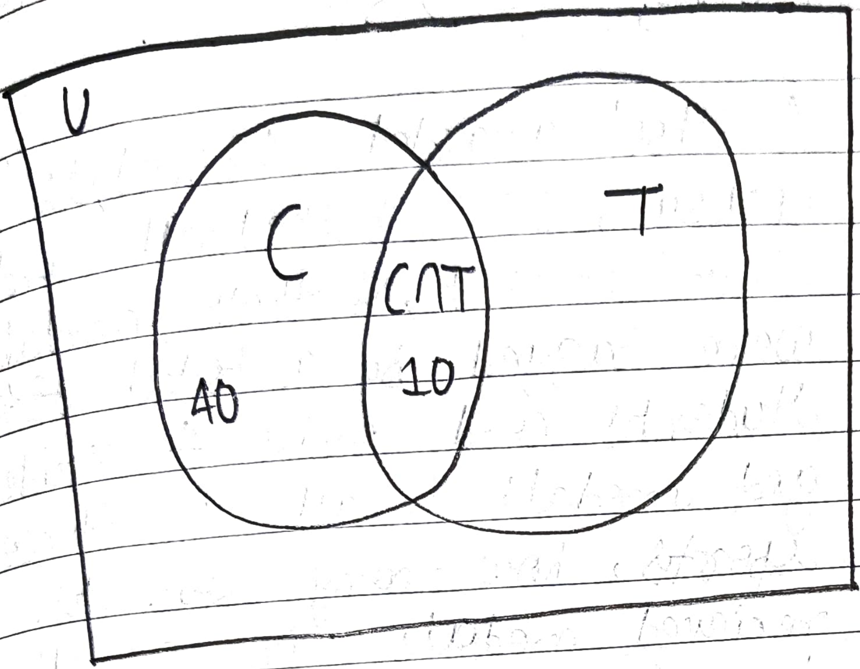
Let us consider,

No. of people who like cricket
$= n(C) = 40$

No. of people who like tennis
$= n(T)$

No. of people who like cricket or tennis $= n(C \cup T) = 65$

No. of people who like cricket and tennis both $= m(C \cap T)$
$= 10$



Now,

$m(C \cap T) = m(C) + m(T) - m(C \cap T)$
$\Rightarrow 65 = 40 + m(T) - 10$
$\Rightarrow m(T) = 65 - 40 + 10$
$= 35.$

Therefore, number of people who like tennis $= 35$

Now,
No. of people who like tennis only
$= m(T - C)$

$m(T-C) = m(T) - m(C \cap T)$
$= 35 - 10$
$= 25.$

Therefore,

the no. of people who like tennis only = 25.

Q. A school awarded 42 medals in hockey, 18 in basketball and 23 in cricket, if these medals were bagged by a total of 65 students and only 4 students got medals in all the three sports, how many students recieved medals in exactly two of the three sports?

Ans: Given:

- Total no. of students = 65
- Medals awarded in Hockey = 42
- Medals awarded in Basketball = 18
- Medals awarded in Cricket = 23
- 4 students got medals in all the three sports.

To find:-

No. of students who received medals in exactly two of the three sports.

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Total number of medals
= Medals awarded in Hockey + Medals awarded in Basketball + Medals awarded in Cricket.

Total number of medals
$= 42 + 28 + 23 = 83$.

It is given that 4 students got medals in all the three sports.

Therefore,

the no. of medals received by those 4 students.
$= 4 \times 3 = 12$

Now,
the no. of medals received by the rest of 61 students.
$= 83 - 12 = 71$.

Among these 61 students, everyone at least received 1 medal.

Therefore, the no. of extra medals
$= 71 - 1 \times 61 = 10$.

Therefore, we can say that 10 students received more than one and less than three medals, or we can say that

10 students received medals in exactly two of three sports.

Q. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, and 3 read all the three newspapers.

Find :-

(i) The number of people who read at least one of the newspapers,

(ii) The number of people who read exactly one newspaper

Ans: Given:

- Total no. of people = 60
- No. of people who read newspaper H = 25
- No. of people who read newspaper T = 26
- No. of people who read newspaper I = 26

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- No. of people who read newspaper H and I both
$= 9$
- No. of people who read newspaper H and T both
$= 11$
- No. of people who read newspaper T and I both
$= 8$

→ No. of people who read all three newspapers
$= 3$

(i) No. of people who read at least one of the newspapers.

Let us consider,
No. of people who read newspaper H
$= n(H) = 25$
No. of people who read newspaper T
$= n(T) = 26$
No. of people who read newspaper I.
$= n(I) = 26$
No. of people who read newspaper H and I both
$= n(H \cap I) = 9$
No. of people who read newspaper H and T both

$= n(H \cap T) = 11.$

No. of people who read newspaper T and I both

$= n(T \cap I) = 8$

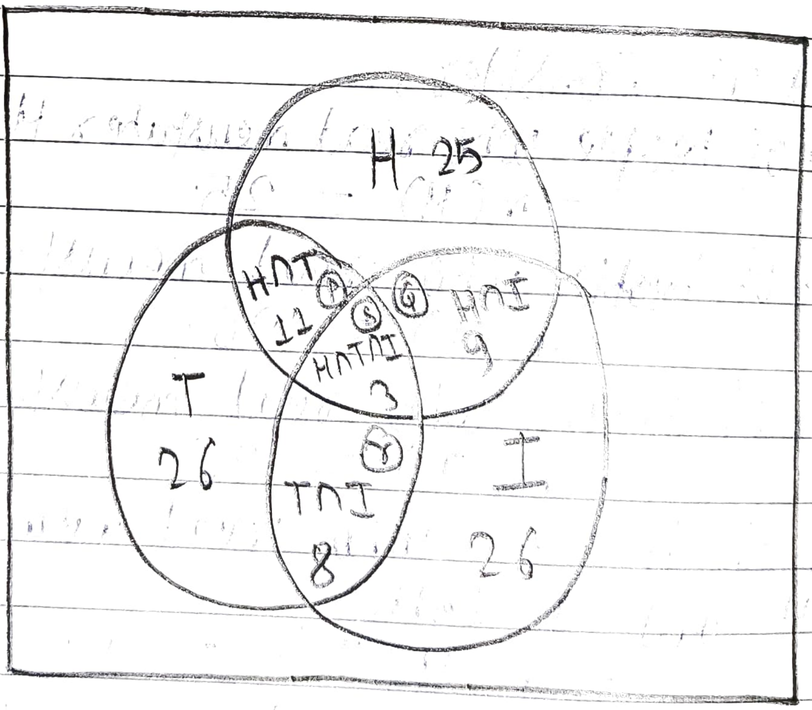
No. of people who read all three newspapers

$= n(H \cap T \cap I)$

$= 3.$

No. of people who read at least one of the three newspapers

$= n(H \cup T \cup I)$



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we know that,

$m(H \cup T \cup I)$
$= m(H) + m(T) + m(I) - m(H \cap I) -$
$m(H \cap T) - m(T \cap I) + m(H \cap T \cap I)$

$= 25 + 26 + 26 - 9 - 11 - 8 + 3$
$= 52.$

Therefore,
Number of people who read at least one of the three newspapers
$= 52.$

(ii) No. of people who read exactly one newspaper.

No. of people who read exactly one newspaper
$= m(H \cup T \cup I) - p - q - r - s$

where,
p = Number of people who read newspaper H and T but not I.
q = Number of people who read newspaper H and I but not T.
r = Number of people who read newspaper T and I but not H.
s = Number of people who read all three newspapers
$= 3.$

$p+s = n(H \cap T)$ ------ (i)

$q+s = n(H \cap I)$ ------ (ii)

$r+s = n(T \cap I)$ ------ (iii)

(i) + (ii) + (iii), we get

$\Rightarrow p+s+q+s+r+s = n(H \cap T) + n(H \cap I) + n(T \cap I)$

$\Rightarrow p+q+r+3s = 9+11+8$

$\Rightarrow p+q+r+s+2s = 28$

$\Rightarrow p+q+r+s+2 \times 3 = 28$

$\Rightarrow p+q+r+s = 28-6$
$= 22$

Now,

$n(H \cup T \cup I) - p - q - r - s$

$= n(H \cup T \cup I) - (p+q+r+s)$

$= 52 - 22$

$= 30$

Hence, 30 people read exactly one newspaper.

Q. In a survey of 100 students, the number of students studying the various languages is found as English only 18; English but not Hindi 23; English and Sanskrit 8; Sanskrit and Hindi 8; English 26; Sanskrit 48 and no language 24. Find :-

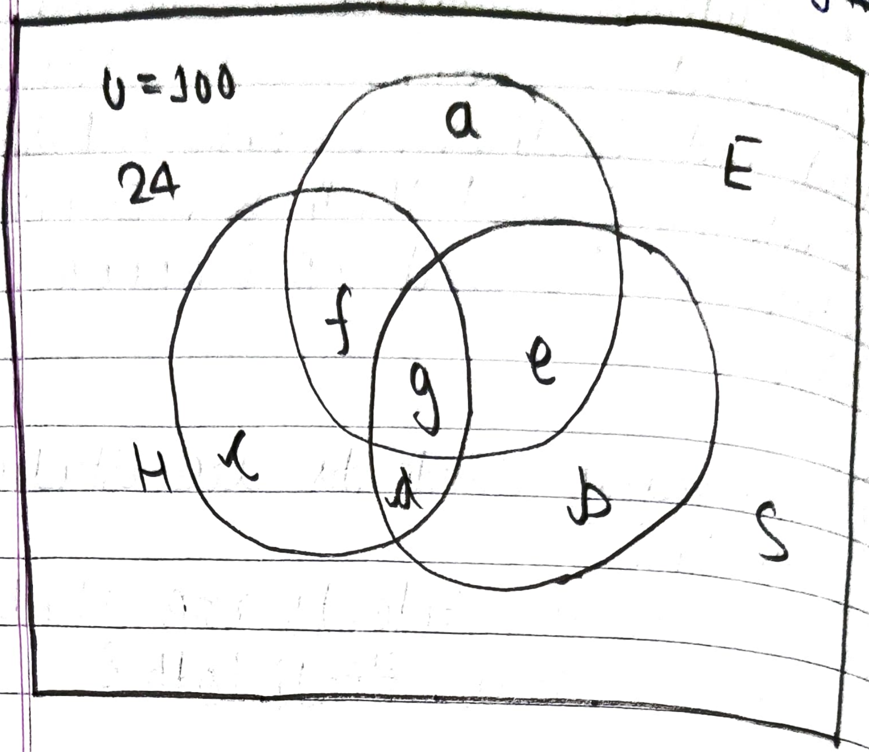
(i.) how many students are studying Hindi?

(ii.) how many students are studying English and Hindi both?

Ans: Given:

- Total no. of students = 100
- Number of students studying English (E) only = 18
- Number of students learning English but not Hindi (H) = 23
- Number of students learning English & Sanskrit (S) = 8
- Number of students learning Sanskrit & Hindi = 8
- Number of students learning English = 26
- Number of students learning Sanskrit = 48
- Number of students learning no language = 24.

(i) No. of students studying Hindi



From the above venn diagram,

a = No. of students who study only English = 18.
b = No. of students who study only Sanskrit
c = No. of students who study only Hindi.
d = No. of students learning Hindi & Sanskrit but not English
e = No. of students learning English & Sanskrit but not Hindi
f = No. of students learning Hindi and English but not Sanskrit
g = No. of students learning all three languages.

$e+g$ = No. of students learning English and Sanskrit = $\cancel{1}8$.
= $n(E \cap S)$.

$g+d$ = No. of students learning hindi and sanskrit.
= $n(H \cap S)$
= 8.

$E = a+e+f+g$
= Number of students learning English

$26 = 18 + \cancel{8}8 + f$
$\Rightarrow f = 26 - 26 = 0$.
Therefore, $f = 0$.

Now,
Number of students learning English but not Hindi
$= a+e = 23$.

$23 = 18 + e$.
$\therefore e = 5$.

Now,
$e+g = 8$.
$5+g = 8 \cancel{-5}$.
$g = 8-5$
$\therefore g = 3$.

$S = b + e + d + g$

= Number of students studying Sanskrit.

$48 = b + 5 + 8$ $(\because d + g = 8)$

$\Rightarrow b = 48 - 13.$

$\therefore b = 35$ (No. of students studying sanskrit only)

also,

$d + g = 8$

$d + 3 = 8$

$\therefore$, $\boxed{d = 5}$

Now,

Number of students studying Hindi only $= c$.

$c = 100 - (a + e + b + d + f + g) - 24$

$= 100 - (18 + 5 + 35 + 5 + 0 + 3) - 24$

$= 100 - 66 - 24$

$= 100 - 94$

$= 10.$

No. of students studying Hindi
$= c+f+g+d$
$= 10+0+3+5$
$= 18.$

Therefore, no. of students ~~student~~ studying Hindi
$= 18.$

(ii) **No. of students studying English & Hindi both.**

Number of students studying English and Hindi both
$= f+g.$
$= 0+3 = 3$

Therefore, No. of students studying English and Hindi both
$= 3.$

Q. In a town of 10,000 families, it was found that 40% of the families buy newspaper A, 20% buy newspaper B, 10% buy newspaper C, 5% buy A and B; 3% buy B and C, and 4% buy A and C. If 2% buy all the three newspapers, find the no. of families which buy :—

(i) A only
(ii) B only
(iii) none of A, B and C.

Ans: Given :

Total number of families = 10000

Percentage of families that buy newspaper A = 40

Percentage of families that buy newspaper B = 20

Percentage of families that buy newspaper C = 10.

Percentage of families that buy newspaper A and B = 5.

Percentage of families that buy newspaper B and C = 3.

Percentage of families that buy newspaper A and C = 4.

Percentage of families that buy all three newspapers = 2.

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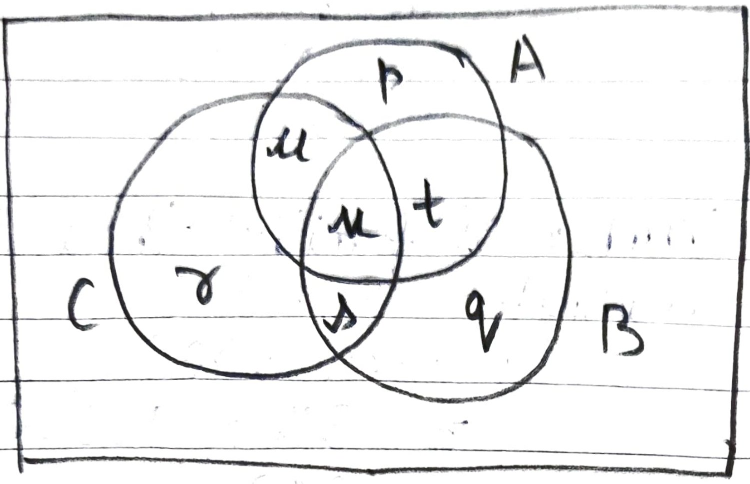
(i.) No. of families that buy newspaper A only



No. of families that buy newspaper A
$= n(A)$
$= 40\%$ of 10000
$= 4000$

No. of families that buy newspaper B
$= n(B)$
$= 20\%$ of 10000
$= 2000$

No. of families that buy newspaper C
$= n(C)$
$= 10\%$ of 10000
$= 1000$

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Number of families that buy newspaper A and B

$= n(A \cap B)$

$= 5\%$ of 10000

$= 500$

Number of families that buy newspaper B and C

$= n(B \cap C)$

$= 3\%$ of 10000

$= 300$

Number of families that buy newspaper A and C.

$= n(A \cap C)$

$= 4\%$ of 10000

$= 400$.

No. of families that buys all three newspaper

$= n(A \cap B \cap C) = v$

$= 2\%$ of 10000

$= 200$.

We have,

$n(A \cap B) = v + t$

$500 = 200 + t$

$t = 500 - 200$

$= 300$

$n(B \cap C) = v + s$
$300 = 200 + s$
$s = 300 - 200$
$= 100$

$n(A \cap C) = v + u$
$400 = 200 + u$
$u = 400 - 200$
$= 200$

p = No. of families that buy newspaper A only.

we have,
$A = p + t + v + u$
$\Rightarrow 4000 = p + 300 + 200 + 200$
$\Rightarrow p = 4000 - 700$
$= 3300$

Therefore,
Number of families that buy newspaper A only $= 3300$

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(ii) No. of families that buy newspaper B only.

q = No. of families that buy newspaper B only.

B = q + s + v + t

2000 = q + 100 + 200 + 300

q = 2000 − 600

= 1400

∴ No. of families that buy newspaper B only = 1400

(iii) No. of families that buys none of the newspaper.

$= 10000 - \{n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)\}$

$= 10000 - (4000 + 2000 + 1000 - 500 - 300 - 400 + 200)$

$= 10000 - 6000$

$= 4000$

∴ No. of families that buy none of newspaper = 4000.

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Q. A class has 175 students. The following description gives the no. students one or more of the subjects in the class: mathematics 100, physics 70, chemistry 46, mathematics and physics 30; mathematics and chemistry 28; physics and chemistry 23; mathematics, physics and chemistry 18.

Find :-

(i) How many students are enrolled in Mathematics alone, physics alone and chemistry alone.

Ans: Given:

- Number of students in class = 175.
- Number of students enrolled in Mathematics = 100
- Number of students enrolled in Physics = 70
- Number of students enrolled in Chemistry = 46.
- Number of students enrolled in Mathematics and Physics = 30.
- Number of students enrolled in Physics and Chemistry = 23
- Number of students enrolled in Mathematics and Physics = 28
- No. of students enrolled in all three subjects = 18.

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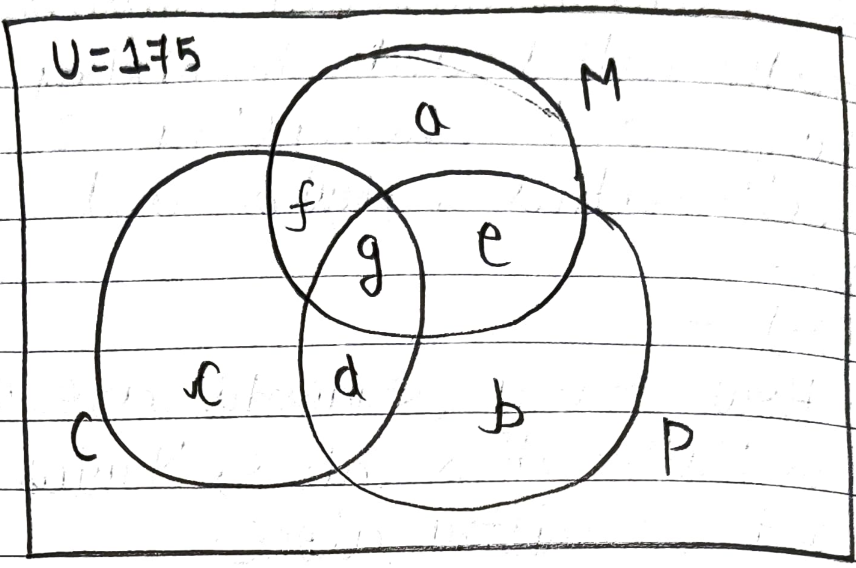
No. of students enrolled in Mathematics alone, Physics alone and chemistry alone.



No. of students enrolled in Mathematics $= 100$
$= n(M)$

No. of students enrolled in Physics $= 70$
$= n(P)$

No. of students enrolled in Chemistry $= 46$
$= n(C)$

No. of students enrolled in Maths & Physics
$= 30 = n(M \cap P)$.

No. of students enrolled in Physics & Chemistry
$= 23 = n(P \cap C)$

No. of students enrolled in Maths & Chemistry
$= 28 = n(M \cap C)$

No. of students enrolled in all the three subjects
$= 18 = n(M \cap P \cap C) = g$

we have,

$n(M \cap P) = e + g$
$30 = e + 18$
$e = 30 - 18 = 12$

$n(M \cap C) = f + g$
$28 = f + 18$
$f = 28 - 18 = 10$

$n(P \cap C) = d + g$
$23 = d + 18$
$d = 23 - 18$
$= 5$

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$a =$ No. of students enrolled only in Mathematics.

$b =$ No. of students enrolled only in Physics = No. of students enrolled only in chemistry.

We have,

$M = a + e + f + g$

$100 = a + 12 + 10 + 18$

$a = 100 - 40$

$a = 60$

Therefore,

No. of students enrolled only in Mathematics $= 60$

$P = b + e + d + g$

$70 = b + 12 + 5 + 18$

$b = 70 - 35$

$b = 35$

therefore,

No. of students enrolled only in Physics $= 35$

$c = e + f + d + g$

$46 = e + 10 + 5 + 18$

$e = 46 - 33$

$= 13$

Therefore,

No. of students enrolled only in chemistry $= 13$.

(ii) Number of students who have not offered any of these subjects.

$= 175 - \{ n(M) + n(P) + n(C) - n(M \cap P) - n(M \cap C) - n(P \cap C) + n(M \cap P \cap C) \}$

$= 175 - (100 + 70 + 46 - 30 - 28 - 23 + 18)$

$= 175 - 153$

$= 22.$

Therefore,

Number of students who have not offered any of these subjects

$= 22.$